5. A clothing shop sells a particular brand of shirt, which comes in three different sizes, small, medium and large.

Each month the manager of the shop orders x small shirts, y medium shirts and z large shirts.

The manager forms constraints on the number of each size of shirts he will have to order.

One constraint is that for every 3 medium shirts he will order at least 5 large shirts.

(a) Write down an inequality, with integer coefficients, to model this constraint.

(2)

Two further constraints are

$$x + y + z \geqslant 250 \text{ and } x \leqslant 0.2(x + y + z)$$

(b) Use these two constraints to write down statements, in context, that describe the number of different sizes of shirt the manager will order.

(3)

The cost of each small shirt is £6, the cost of each medium shirt is £10 and the cost of each large shirt is £15

The manager must minimise the total cost of all the shirts he will order.

(c) Write down the objective function.

(1)

Initially, the manager decides to order exactly 150 large shirts.

(d) (i) Rewrite the constraints, as simplified inequalities with integer coefficients, in terms of x and y only.

(ii) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region R.

(6)

(e) Use the objective line method to find the optimal vertex, V, of the feasible region. You must make your objective line clear and label V.

(2)

(f) Write down the number of each size of shirt the manager should order. Calculate the total cost of this order.

(2)

Later, the manager decides to order exactly 50 small shirts and exactly 75 medium shirts instead of 150 large shirts.

(g) Find the minimum number of large shirts the manager should order and show that this leads to a lower cost than the cost found in (f).

(2)

(Total 18 marks)