7. A school is planning to run several training days next year for its new teachers, middle leaders and senior leaders.

Next year, the school will need to run

- at least 20 training days in total,
- at most twice as many training days for new teachers when compared to the total number of training days required for both middle and senior leaders,
- at most 25% of the training days for senior leaders.

The costs of running a training day for new teachers, middle leaders and senior leaders are £400, £550 and £750 respectively.

The school wants to minimise the total cost of running the training days.

Let x be the number of training days required for new teachers.
Let y be the number of training days required for middle leaders.
Let z be the number of training days required for senior leaders.

(a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(6)

The school decides that the number of training days for middle leaders and the number of training days for senior leaders should be in the ratio 5 : 3
This reduces two of the constraints to $5x \leqslant 16y$ and $4y \leqslant 5x$.

(b) (i) Express the constraint representing the requirement for a total of at least 20 training days as a simplified inequality with integer coefficients in terms of x and y only.

(ii) Express the objective in the form $ax + by$ where a and b are integers.

(3)

(c) Represent the constraints in x and y only on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4)

(d) Use the objective line method to locate the optimal vertex, V, of the feasible region. You must make your objective line clear and label the optimal vertex, V.

(3)

(e) Hence, determine

(i) the total cost of running the training days,

(ii) the number of training days required for senior leaders.

(2)

(Total 18 marks)

TOTAL FOR PAPER: 75 MARKS

END