7. A company makes three types of storage container, small, medium and large.

The company owner knows that each week she should make

- at least 40 containers in total
- at least twice as many large containers as medium containers
- at most 60% small containers

Each small container requires 1 hour to make, each medium container requires 1.5 hours to make, and each large container requires 2.5 hours to make. The company has a total of 75 hours per week available to make all the containers.

Each small container costs £9 to make, each medium container costs £12 to make and each large container costs £16 to make.

The company owner wants to minimise her total cost.

- Let x represent the number of small containers made
- Let y represent the number of medium containers made
- Let z represent the number of large containers made

(a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(7)

The company owner now decides to make exactly 45 containers.

(b) Explain why the minimum total cost is achieved when $7x + 4y$ is maximised.

(3)

The requirement to make exactly 45 containers reduces the constraints of the problem to the following:

$$x + 3y \leqslant 45$$

$$0 \leqslant x \leqslant 27$$

$$3x + 2y \geqslant 75$$

$$y \geqslant 0$$

(c) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4)

(d) Use the objective line method to find the optimal vertex, V, of the feasible region. You must make your objective line clear and label V.

(2)

(e) Write down the number of each type of container the company should make. Calculate the corresponding total cost.

(2)

(Total 18 marks)

TOTAL FOR PAPER = 75 MARKS

END