Question Number	Scheme	Marks
7. (a)	Prim: AH, FH, EH, FG, DG, CG, BC	M1 A1 A1 **(3)**
(b)	Initial upper bound 2(201) = 402 (km)	B1ft **(1)**
(c)	A – H – F – G – D – B – C – E – A 27 + 28 + 31 + 29 + 32 + 26 + 38 + 37 = 248	M1 A1 **(2)**
(d)	Nearest neighbour starting at E has a length of $212 + x$ As $x \leqslant 35 \Rightarrow$ the NN route starting at E is at most 247 (km) and therefore the NN starting at E gives the better upper bound as it is less than the one starting at A (which was 248 (km))	M1 A1 **(2)**
(e)	Lower bound is given by $(201-27)+27+x=235$	M1
	$x=34$	A1
	$235 \leqslant$ optimal length $\leqslant 246$	M1 A1 **(4)**
		12 marks

Notes for Question 7

a1M1: Prim's – first three arcs correctly chosen in order (AH, FH, EH, …) **or** first four nodes {A, H, F, E, …} correctly chosen in order. If any explicit rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, -, - , 4 , 3, -, 2} so check there too. Starting at any other node apart from A can score M1 only for first three arcs chosen correctly

a1A1: First five arcs correctly chosen in order (AH, FH, EH, FG, DG, …) **or** all eight nodes {A, H, F, E, G, D, C, B} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 8, 7, 6, 4, 3, 5, 2} (**no** missing numbers)

a2A1: CSO – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

b1B1ft: Follow through double the stated length of their MST

c1M1: NN starting at A – must have at least A – H – F – G – D –… allow if stated in terms of arcs

c2A1: CAO on length (248) **and** route (must return to A but can be stated in terms of arcs)

d1M1: Calculating the correct length of the NN route starting at E ($212 + x$) **and** attempting to use the range of values for x to determining the better upper bound (implied by 247 seen or [244, 247]).

d1A1: Correct best upper bound stated (the one starting at E) together with a correct comparison of **248** (possibly implicit - if this value is not explicitly stated in (d) then 248 must have been seen in (c)) with 247 (or an indication of 'at most' 247). **For those who obtained an answer of 211 in (c) and say that $212 + x$ is always bigger (without using the given interval for x to find the UB) then no marks in this part**

e1M1: Correct method for calculating x (which is the weight of MST from (a)/(b) – 27 + two smallest arcs incident to A (the 27 and x) equal to 235). If using the doubled value from (b) then **M0**. If not using the weight of the MST from (a) then they must be using either 174 **or** 26 + 30 + 29 + 31 + 28 + 30 **or** explicitly using the correct six arcs only (BC, CG, GD, GF, FH, HE) so not just circled in one of the tables. The correct value of x (with either no working or no incorrect working) clearly stated can imply this mark

e1A1: CAO for x (34) – **as a minimum** must have seen the calculation $201+x=235$ to award this mark

e2M1: Any indication of an interval from 235 to either 246, 247 or 248 (this mark is **not** dependent on the previous M mark)

e2A1: CAO (condone $235 <$ optimal length $\leqslant 246$ and allow equivalent interval notation e.g. (235, 246] or [235, 246]) – this mark is dependent on all previous marks in (e) (so must have found that x equals 34).
The correct interval (with no others) with no supporting working scores **M0A0M1A0**
The minimum requirement for full marks is: $201 + x = 235 \Rightarrow x = 34 \therefore [235, 246]$
For those who simply state $x = 34$ (only) followed by the correct interval they score **M1A0M1A0**