7.

	A	B	C	D	E	F	G	H
A	–	38	37	x	37	42	41	27
B	38	–	26	32	33	38	37	34
C	37	26	–	39	38	39	30	39
D	x	32	39	–	37	36	29	36
E	37	33	38	37	–	32	33	30
F	42	38	39	36	32	–	31	28
G	41	37	30	29	33	31	–	33
H	27	34	39	36	30	28	33	–

The network represented by the table shows the least distances, in km, between eight museums, A, B, C, D, E, F, G and H.

A tourist wants to visit each museum at least once, starting and finishing at A. The tourist wishes to minimise the total distance travelled. The shortest distance between A and D is x km where $32 \leqslant x \leqslant 35$

(a) Using Prim's algorithm, starting at A, obtain a minimum spanning tree for the network. You must clearly state the order in which you select the arcs of your tree.

(3)

(b) Use your answer to (a) to determine an initial upper bound for the length of the tourist's route.

(1)

(c) Starting at A, use the nearest neighbour algorithm to find another upper bound for the length of the tourist's route. Write down the route that gives this upper bound.

(2)

The nearest neighbour algorithm starting at E gives a route of

E – H – A – D – G – C – B – F – E

(d) State which of these two nearest neighbour routes gives the better upper bound. Give reasons for your answer.

(2)

Starting by deleting A, and all of its arcs, a lower bound of 235 km for the length of the route is found.

(e) Determine the smallest interval that must contain the optimal length of the tourist's route. You must make your method and working clear.

(4)

(Total for Question 7 is 12 marks)