7. A farmer has 100 acres of land available that can be used for planting three crops: A, B and C.

It takes 2 hours to plant each acre of crop A, 1.5 hours to plant each acre of crop B and 45 minutes to plant each acre of crop C. The farmer has 138 hours available for planting.

At least one quarter of the total crops planted must be crop A.

For every three acres of crop B planted, at most five acres of crop C will be planted.

The farmer expects a profit of £160 for each acre of crop A planted, £75 for each acre of crop B planted and £125 for each acre of crop C planted.

The farmer wishes to maximise the profit from planting these three crops.

Let x, y and z represent the number of acres of land used for planting crop A, crop B, and crop C respectively.

(a) Formulate this information as a linear programming problem. State the objective, and list the constraints as simplified inequalities with integer coefficients.

(6)

The farmer decides that all 100 acres of available land will be used for planting the three crops.

(b) Explain why the maximum total profit is achieved when $-7x + 10y$ is minimised.

(2)

The farmer's decision to use all 100 acres reduces the constraints of the problem to the following:

$$x \geqslant 25$$

$$3x + 8y \geqslant 300$$

$$x + y \leqslant 100$$

$$5x + 3y \leqslant 252$$

$$y \geqslant 0$$

(c) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4)

(d) (i) Determine the exact coordinates of each of the vertices of R.

(ii) Apply the vertex method to determine how the 100 acres should be used for planting the three crops.

(iii) Hence find the corresponding maximum expected profit.

(5)

(Total for Question 7 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS