5: A gardener wants to plant some rose bushes. She will use three different colours, red, yellow and white.
The gardener decides to use linear programming to determine how many of each colour she will plant.

Let x represent the number of red rose bushes, y represent the number of yellow rose bushes and z represent the number of white rose bushes.

The gardener wants to maximise the total number of rose bushes she will plant.

(a) Write down the objective for this problem.

(1)

The gardener has a total of £1400 available to spend. Given that

- red rose bushes cost £12.50 each
- yellow rose bushes cost £10 each
- white rose bushes cost £15 each

(b) show that one constraint is given by

$$5x + 4y + 6z \leqslant 560$$

(2)

The gardener decides that

- the number of red rose bushes must be at least 40% of the total number of rose bushes
- there must be at least three white rose bushes for every two yellow rose bushes

(c) Use this information to formulate two further constraints, which should be fully simplified with integer coefficients.

(3)

The gardener decides to plant exactly 30 white rose bushes. This reduces two of the constraints to

$$5x + 4y \leqslant 380$$
$$y \leqslant 20$$

(d) Show that the third constraint can be reduced to

$$3x - 2y \geqslant 60$$

(2)

(e) Draw these three constraints using the axes in the answer book and hence determine the number of red rose bushes and the number of yellow rose bushes that the gardener will plant.

(4)

(Total for Question 5 is 12 marks)