4.

	A	B	C	D	E	F	G	H
A	–	34	42	29	36	43	28	$6x + 7$
B	34	–	27	40	28	51	43	$8x - 2$
C	42	27	–	31	41	37	32	46
D	29	40	31	–	25	34	43	39
E	36	28	41	25	–	30	38	52
F	43	51	37	34	30	–	31	48
G	28	43	32	43	38	31	–	34
H	$6x + 7$	$8x - 2$	46	39	52	48	34	–

The table shows the shortest distances, in miles, between eight towns, A, B, C, D, E, F, G and H, where x is an unknown positive integer.

Raphael wants to visit all eight towns and needs to find a route that minimises the total distance travelled returning to the starting point.

An initial upper bound for the length of Raphael's route, obtained using the nearest neighbour algorithm starting at H, is 256 miles. This nearest neighbour route starts with arc **HG**.

(a) Use the nearest neighbour algorithm starting at A, to obtain an expression for a second upper bound for the length of Raphael's route. You must state the route and its length in terms of x.

(3)

Given that the upper bound obtained by starting at A is better than the initial upper bound,

(b) use the answer to part (a) to calculate the value of x

(3)

(c) By deleting A and all its arcs, obtain a lower bound for the length of Raphael's route.

(3)

(d) Write down the smallest interval that must contain the length of Raphael's route.

(1)

(Total for Question 4 is 10 marks)