7.

	A	B	C	D	E	F	G	H
A	–	43	38	52	44	39	57	48
B	43	–	41	39	50	46	36	43
C	38	41	–	48	53	44	51	45
D	52	39	48	–	43	52	47	49
E	44	50	53	43	–	50	42	46
F	39	46	44	52	50	–	48	54
G	57	36	51	47	42	48	–	42
H	48	43	45	49	46	54	42	–

The table above shows the shortest distance, in miles, between eight towns, A, B, C, D, E, F, G and H.

A delivery driver must visit each town at least once and return to the starting point. The driver wishes to minimise the total distance travelled.

(a) (i) Using Prim's algorithm, starting at **D**, obtain a minimum spanning tree for the network. You must clearly state the order in which you select the arcs of your tree.

(ii) State the weight of the tree.

(3)

(b) Draw the minimum spanning tree on the diagram in the answer book.

(1)

(c) Use your answer to part (a) to determine an initial upper bound for the length of the delivery driver's route.

(1)

(d) Use one shortcut, starting from **B**, to obtain an improved upper bound with a length less than 500 miles, stating the shortcut used and the length of the improved upper bound.

(2)

(e) Show that there are two nearest neighbour routes starting from **A**, stating both routes and their lengths.

(3)

Question 7 continued

(f) By deleting **H** and all of its arcs, calculate a lower bound for the length of the delivery driver's route.

(2)

A second lower bound of 321 miles is calculated by deleting **A** and all of its arcs.

(g) Write down the smallest interval that must contain the length of the delivery driver's route.

(1)

(Total for Question 7 is 13 marks)