| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **5.** | $H: xy = 16;\ P\left(4p, \frac{4}{p}\right),\ p \neq 0,$ lies on $H$.<br>Tangent to $H$ at $P$ passes through the point $(7, 1)$ | | |
| (a) | $y = \frac{16}{x} = 16x^{-1} \Rightarrow \frac{dy}{dx} = -16x^{-2}$ or $-\frac{16}{x^2}$ | $\frac{dy}{dx} = \pm k x^{-2};\ k \neq 0$ | M1 |
| | $xy = 16 \Rightarrow x\frac{dy}{dx} + y = 0$ | Uses implicit differentiation to give $\pm x\frac{dy}{dx} \pm y$ | |
| | $x = 4t,\ y = \frac{4}{t} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\left(\frac{4}{t^2}\right)\left(\frac{1}{4}\right)$ | their $\frac{dy}{dt} \times \frac{1}{\text{their } \frac{dy}{dt}}$; **Condone** $t \equiv p$ | |
| | So at $P$, $m_T = -\frac{1}{p^2}$ | Correct calculus work leading to $m_T = -\frac{1}{p^2}$ | A1 |
| | • $y - \frac{4}{p} = -\frac{1}{p^2}(x - 4p)$ **or**<br>• $\frac{4}{p} = -\frac{1}{p^2}(4p) + c \Rightarrow y = -\frac{1}{p^2}x +$ their $c$ | Correct straight line method for an equation of the tangent where $m_T\left(\neq \frac{-1}{\text{their } m_T} \text{ or } \neq \frac{1}{\text{their } m_T}\right)$ is found by using calculus.<br>**Note:** $m_T$ must be a function of $p$<br>**Note:** Condone (slip) of using $m_T = -(\text{their } m_T)$ | M1 |
| | Correct algebra leading to $x + p^2 y = 8p$ * | Correct solution only | A1 * |
| | | | **(4)** |
| (b) | $\{(7, 1) \Rightarrow\}\ \ 7 + p^2 = 8p$ | Substitutes $x = 7,\ y = 1$ into the given equation or their answer to part (a).<br>**Note:** Condone substituting $x = 1,\ y = 7$ into the given equation or their answer to part (a) for M1 | M1 |
| | $\{\Rightarrow p^2 - 8p + 7 = 0\}$ | | |
| | $(p - 7)(p - 1) = 0 \Rightarrow p = ...$ | **dependent on the previous M mark**<br>Correct method (e.g. factorising, applying the quadratic formula or completing the square) of solving a 3TQ to find $p = ...$ | dM1 |
| | $\{p = 1 \Rightarrow\}\ x = 4,\ y = 4$<br>$\{p = 7 \Rightarrow\}\ x = 28,\ y = \frac{4}{7}$ or awrt 0.57 | **dependent on substituing $x = 7, y = 1$ into the given equation or their answer to part (a)**<br>Obtains at least one correct set of corresponding values for $x = ...$ **and** $y = ...$ | A1 |
| | {So $P$ can be} $(4, 4), \left(28, \frac{4}{7}\right)$ | Both correct sets of coordinates of $B$ | A1 |
| | | | **(4)** |
| | | | **8** |

| | | Question 5 Notes |
|---|---|---|
| **5.** (a) | **Note** | Allow $yp^2 + x = 8p$ or $8p = x + p^2y$ or $8p = p^2y + x$ for the final A1 |
| (b) | **Note** | Do not confuse $(7, 1)$ or $x = 7,\ y = 1$ with $p = 7, 1$ |
| | **Note** | A decimal answer of e.g. $(4, 4), (28, 0.57)$ (without a correct exact answer) is 2nd A0 |
| | **Note** | Imply the dM1 mark for ***writing down the correct*** roots for ***their*** quadratic equation. .<br>E.g. $7 + p^2 = 8p$ or $p^2 - 8p + 7 = 0 \rightarrow p = 7, 1$ |
| | **Note** | E.g. give dM0 for $7 + p^2 = 8p$ or $p^2 - 8p + 7 = 0 \rightarrow p = -7, -1$ [incorrect solution] with NO INTERMEDIATE working. |
| | **Note** | Give M1 dM1 A1 for either<br>• $7 + p^2 = 8p \rightarrow x = 4,\ y = 4$ or $(4, 4)$<br>• $7 + p^2 = 8p \rightarrow x = 28,\ y = \frac{4}{7}$ or awrt 0.57 **or** $\left(28, \frac{4}{7}\right)$ **or** $(28,$ awrt $0.57)$<br>with NO INTERMEDIATE working. |
| | **Note** | Give M1 dM1 A1 A1 for<br>• $7 + p^2 = 8p \rightarrow (4, 4), \left(28, \frac{4}{7}\right)$<br>with NO INTERMEDIATE working. |
| | **Note** | Give M0 dM0 A0 A0 for writing down $(4, 4), \left(28, \frac{4}{7}\right)$ with no prior working. |
| | **Note** | Only a maximum of M1 dM1 A0 A0 can be scored for<br>substituting for $x = 1,\ y = 7$ (and not $x = 7,\ y = 1$) into $x + p^2y = 8p$<br>**Note:** $x = 1,\ y = 7 \Rightarrow 1 + 7p^2 = 8p \Rightarrow (7p - 1)(p - 1) \Rightarrow p = \frac{1}{7}, 1 \Rightarrow \left(\frac{4}{7}, 28\right), (4, 4)$ |
| | **Note** | **<u>Alt 1 Method</u>**<br>• $x = 7,\ y = 1 \Rightarrow 7 + p^2 = 8p \Rightarrow (p - 1)(p - 7) \Rightarrow p = 1, 7$<br>• $p = 1 \Rightarrow x + (1)y = 8(1)$ and $x + \frac{16}{x} = 8 \Rightarrow x^2 - 8x + 16 = 0 \Rightarrow (x - 4)(x - 4) = 0$<br>$\Rightarrow x = 4,\ y = 4 \Rightarrow (4, 4)$<br>• $p = 7 \Rightarrow x + 49y = 56$ and $x + 49\left(\frac{16}{x}\right) = 56 \Rightarrow x^2 - 56x + 784 = 0 \Rightarrow (x - 28)(x - 28) = 0$<br>$\Rightarrow x = 28,\ y = \frac{4}{7} \Rightarrow \left(28, \frac{4}{7}\right)$ |
| | **Note** | **<u>Incorrect method of substituting $xy = 16$ and $(7, 1)$ into $x + p^2y = 8p$</u>**<br>Give M0 dM0 A0 A0 for<br>• $x + p^2\left(\frac{16}{x}\right) = 8p$ and $x = 7 \Rightarrow 7 + \frac{16}{7}p^2 = 8p \Rightarrow 16p^2 - 56p + 49 = 0 \Rightarrow (4p - 7)(4p - 7) = 0$<br>$\Rightarrow p = \frac{7}{4} \Rightarrow x = 7,\ y = \frac{16}{7} \Rightarrow \left(7, \frac{16}{7}\right)$<br>• $\frac{16}{y} + p^2y = 8p$ and $y = 1 \Rightarrow 16 + p^2 = 8p \Rightarrow p^2 - 8p + 16 = 0 \Rightarrow (p - 4)(p - 4) = 0$<br>$\Rightarrow p = 4 \Rightarrow x = 16,\ y = 1 \Rightarrow (16, 1)$ |
| | **Note** | Give M1 dM0 A0 A0 for<br>• $x = 7,\ y = 1$ into $x + p^2y = 8p \Rightarrow 7 + p^2 = 8 \Rightarrow (p + 1)(p - 1) \Rightarrow p = 1, -1 \Rightarrow (4, 4), (-4, -4)$ |