Question Number	Scheme	Notes	Marks
7.	$\mathbf{P}=\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$; (a) $\mathbf{P}^3=8\mathbf{I}$; (c) $\mathbf{P}^{35}=2^k\begin{pmatrix}-1 & a\\ b & -1\end{pmatrix}$		
(a)	$\{\mathbf{P}^2=\}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}=\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$	Finds $\mathbf{P}^2$ (which can be un-simplified) with at least 3 correct elements for $\mathbf{P}^2$	M1
	$\{\mathbf{P}^3=\}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}=\begin{pmatrix}8 & 0\\ 0 & 8\end{pmatrix}=8\mathbf{I}$ * **or** $\{\mathbf{P}^3=\}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}=\begin{pmatrix}8 & 0\\ 0 & 8\end{pmatrix}=8\mathbf{I}$ *	**dependent on the previous M mark** Multiplies $\mathbf{P}^2$ by $\mathbf{P}$ or multiplies $\mathbf{P}$ by $\mathbf{P}^2$ to give a 2×2 matrix of 4 elements for $\mathbf{P}^3$ with at least 2 correct elements	dM1
		Correct proof with no errors	A1 *
			(3)
(b)	Enlargement	Enlargement or enlarge or dilation	M1
	Centre $(0,0)$ with scale factor 2	about $(0,0)$ or about O or about the origin **and** scale or factor or times **and** 2	A1
	Rotation	Rotation or rotate (condone turn)	M1
	120 degrees (anticlockwise) about $(0,0)$	**Both** 120 degrees or $\frac{2\pi}{3}$ or 240 degrees clockwise or $\frac{4\pi}{3}$ clockwise **and** about $(0,0)$ or about O or about the origin	A1
			(4)
(c) **Way 1**	$\mathbf{P}^{35}=(\mathbf{P}^3)^{11}\times\mathbf{P}^2$ **or** $\mathbf{P}^{35}=\mathbf{P}^{33}\times\mathbf{P}^2$		
	$=(8\mathbf{I})^{11}\times\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$ **or** $=(2\mathbf{I})^{33}\times\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$	$((8\mathbf{I})^{11}$ or $(8)^{11})\times$(their $\mathbf{P}^2$) or $((2\mathbf{I})^{33}$ or $(2)^{33})\times$(their $\mathbf{P}^2$)	M1
	$=2^{34}\begin{pmatrix}-1 & \sqrt{3}\\ -\sqrt{3} & -1\end{pmatrix}$	Correct answer **Note:** $k=34$, $a=\sqrt{3}$, $b=-\sqrt{3}$	A1
			(2)
(c) **Way 2**	$\mathbf{P}^{35}=(\mathbf{P}^3)^{12}\times\mathbf{P}^{-1}$ **or** $\mathbf{P}^{35}=\mathbf{P}^{36}\times\mathbf{P}^{-1}$		
	$=(8\mathbf{I})^{12}\times\frac{1}{(-1)(-1)-(-\sqrt{3})(\sqrt{3})}\begin{pmatrix}-1 & \sqrt{3}\\ -\sqrt{3} & -1\end{pmatrix}$ **or** $=(2\mathbf{I})^{36}\times\frac{1}{(-1)(-1)-(-\sqrt{3})(\sqrt{3})}\begin{pmatrix}-1 & \sqrt{3}\\ -\sqrt{3} & -1\end{pmatrix}$	$((8\mathbf{I})^{12}$ or $(8)^{12})\times\frac{1}{\text{their det}(\mathbf{P})}\begin{pmatrix}-1 & \sqrt{3}\\ -\sqrt{3} & -1\end{pmatrix}$ or $((2\mathbf{I})^{36}$ or $(2)^{36})\times\frac{1}{\text{their det}(\mathbf{P})}\begin{pmatrix}-1 & \sqrt{3}\\ -\sqrt{3} & -1\end{pmatrix}$ where their det$(\mathbf{P})>1$	M1
	$\left\{=\left(2^{36}\right)\left(\frac{1}{4}\right)\begin{pmatrix}-1 & \sqrt{3}\\ -\sqrt{3} & -1\end{pmatrix}\right\}=2^{34}\begin{pmatrix}-1 & \sqrt{3}\\ -\sqrt{3} & -1\end{pmatrix}$	Correct answer **Note:** $k=34$, $a=\sqrt{3}$, $b=-\sqrt{3}$	A1
			(2)
			9

	Question 7 Notes	
7. (a)	**Note**	Proof must contain the final steps of $= \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ **and** $= 8\mathbf{I}$ **or** $= \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ **and** = RHS
	Note	Other acceptable proofs for M1 dM1 A1 include • $\mathbf{P}^3 = \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$ or $\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}^3$ $= \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = 8\mathbf{I}$ * • $\mathbf{P}^3 = \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$ or $\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}^3$ $= \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = 8\mathbf{I}$ * • $\mathbf{P}^3 = \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = 8\mathbf{I}$ * • $\mathbf{P}^3 = \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = 8\mathbf{I}$ *
(b)	**Note**	"original point" is not acceptable in place of the word "origin".
	Note	"expand" is 1st M0
	Note	"enlarge x by 2 and no change in y" is 1st M0 1st A0
	Note	Writing "120 degrees" by itself implies by convention "120 degrees anti-clockwise". So • "Rotation 120 degrees about O" is 2nd M1 2nd A1 • "Rotation 120 degrees clockwise about O" is 2nd M1 2nd A0
	Note	Writing down "centre $(0, 0)$ with scale factor 2" with no reference to "enlargement" or "enlarge" or "dilation" is 1st M0 1st A0
	Note	Writing down "120 degrees anti-clockwise about O" with no reference to "rotation" or "turn" is 2nd M0 2nd A0
	Note	Give 1st M1 1st A0 for writing "stretch parallel to x-axis and y-axis"
	Note	Give 1st M1 1st A0 for writing "stretch scale factor 2 parallel to x-axis and stretch scale factor 2 parallel to y-axis {with centre $(0, 0)$ }"
	Note	If a candidate would score M1 A1 M1 A1 in part (b) and there is an error in their solution (e.g. a third transformation given) then give M1 A1 M1 A0
(c)	**Note**	$8^{11} = 2^{33} = 8589934592$
	Note	$8^{12} = 2^{36} = 68719476736$
	Note	(their $\mathbf{P}^2$) must be a genuine attempt at $\mathbf{P}^2$ or must be for (their $\mathbf{P}^2$) seen in part (a)
	Note	Allow M1 A1 for writing $\mathbf{P}^{35} = 2^{34}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$
	Note	Stating $k = 34$, $a = \sqrt{3}$, $b = -\sqrt{3}$ from no working is M1 A1
	Note	Give M0 A0 for $\mathbf{P}^4 = 2^3\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \Rightarrow \mathbf{P}^{35} = 2^{34}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$

	Question 7 Notes Continued	
7. (c)	**Note**	Writing down $(8\mathbf{I})^{11}\times\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$ or $(2\mathbf{I})^{33}\times\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$ or $(8\mathbf{I})^{11}\times\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}^2$ or $(2\mathbf{I})^{33}\times\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}^2$ **with no attempt** to evaluate $\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$ is M0
	Note	Allow M1 for **applying** $\mathbf{P}^{35}=(\mathbf{P}^3)^{11}\times\mathbf{P}^2$ or $\mathbf{P}^{35}=\mathbf{P}^{33}\times\mathbf{P}^2$ E.g. Allow M1 for $\begin{pmatrix}8 & 0\\ 0 & 8\end{pmatrix}^{11}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$ or $\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}^{33}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$ or $\begin{pmatrix}8^{11} & 0\\ 0 & 8^{11}\end{pmatrix}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$ or $\begin{pmatrix}2^{33} & 0\\ 0 & 2^{33}\end{pmatrix}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$ or $(8)^{11}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$ or $(2)^{33}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$
	Note	Allow M1 for $(2)^{35}\begin{pmatrix}\cos 240 & -\sin 240\\ \sin 240 & \cos 240\end{pmatrix}$ **or** $(2)^{35}\begin{pmatrix}\cos 4200 & -\sin 4200\\ \sin 4200 & \cos 4200\end{pmatrix}$ **or** $(2)^{35}\begin{pmatrix}-0.5 & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -0.5\end{pmatrix}$ **or** equivalent in radians
	Note	Give M0 for $\mathbf{P}^{35}=(\mathbf{P}^3)^{11}\times\mathbf{P}^2$ by itself
	Note	Give M0 for $\mathbf{P}^{35}=\mathbf{P}^{33}\times\mathbf{P}^2$ by itself