Question Number	Scheme	Notes	Marks
8.	(i) $\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^n = \begin{pmatrix}1+4n & -8n\\ 2n & 1-4n\end{pmatrix}$	(ii) $u_1 = 8,\ u_2 = 40,\ u_{n+2} = 8u_{n+1} - 12u_n \Rightarrow u_n = 6^n + 2^n$	
(i)	$n=1,\ \text{LHS} = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$, $\text{RHS} = \begin{pmatrix}1+4(1) & -8(1)\\ 2(1) & 1-4(1)\end{pmatrix} = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$	Shows or states that **either** $\text{LHS} = \text{RHS} = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$ **or** $\text{LHS} = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix},\ \text{RHS} = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$	B1
	(Assume the result is true for $n=k$)		
	$\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^{k+1} = \begin{pmatrix}1+4k & -8k\\ 2k & 1-4k\end{pmatrix}\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$ **or** $= \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}\begin{pmatrix}1+4k & -8k\\ 2k & 1-4k\end{pmatrix}$	States intention to multiply $\begin{pmatrix}1+4k & -8k\\ 2k & 1-4k\end{pmatrix}$ by $\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$ **(either way round)**	M1
	$= \begin{pmatrix}5+20k-16k & -8-32k+24k\\ 10k+2-8k & -16k-3+12k\end{pmatrix}$ **or** $= \begin{pmatrix}5+20k-16k & -40k-8+32k\\ 2+8k-6k & -16k-3+12k\end{pmatrix}$ **or** $= \begin{pmatrix}5+4k & -8-8k\\ 2+2k & -4k-3\end{pmatrix}$	**dependent on the previous M mark** Multiplies out to give a correct un-simplified matrix with at least 3 correct elements	dM1
	$= \begin{pmatrix}1+4(k+1) & -8(k+1)\\ 2(k+1) & 1-4(k+1)\end{pmatrix}$	***Uses algebra*** to achieve this result with no errors	A1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 **cso**
			(5)
(ii)	$\{n=1,\}\ u_1 = 6^1 + 2^1 = 8$; $\{n=2,\}\ u_2 = 6^2 + 2^2 = 40$	Shows $u_1 = 8$ by writing an intermediate step of e.g. $6^1 + 2^1$ or $6+2$ **and** shows $u_2 = 40$ by writing an intermediate step of e.g. $6^2 + 2^2$ or $36+4$	B1
	(Assume the result is true for $n=k$ and $n=k+1$)		
	$\{u_{k+2} = 8u_{k+1} - 12u_k \Rightarrow\}$ $u_{k+2} = 8(6^{k+1} + 2^{k+1}) - 12(6^k + 2^k)$	Finds u_{k+2} by attempting to substitute $u_{k+1} = 6^{k+1} + 2^{k+1}$ and $u_k = 6^k + 2^k$ into $u_{k+2} = 8u_{k+1} - 12u_k$ Condone one slip	M1
	either $\{u_{k+2}\} = 48(6^k) + 16(2^k) - 12(6^k + 2^k)$ $= 36(6^k) + 4(2^k)$ $= 6^2(6^k) + 2^2(2^k)$	Expresses u_{k+2} correctly in terms of **only** 6^k **and** 2^k **or only** 6^{k+1} **and** 2^{k+1} **or** as $8(6^{k+1}) - 2(6^{k+1}) + 4(2^{k+2}) - 3(2^{k+2})$ **or** as $48(6^k) - 12(6^k) + 4(2^{k+2}) - 3(2^{k+2})$	A1 (M1 on ePEN)
	or $\{u_{k+2}\} = 8(6^{k+1} + 2^{k+1}) - 2(6^{k+1}) - 6(2^{k+1})$ $= 6(6^{k+1}) + 2(2^{k+1})$		
	or $\{u_{k+2}\} = 8(6^{k+1}) - 2(6^{k+1}) + 4(2^{k+2}) - 3(2^{k+2})$		
	or $\{u_{k+2}\} = 48(6^k) - 12(6^k) + 4(2^{k+2}) - 3(2^{k+2})$		
	$= 6^{k+2} + 2^{k+2}$	**dependent on the previous A mark** ***Uses algebra*** in a complete method to achieve this result with no errors	A1
	If the result is true for $n=k$ and for $n=k+1$, then it is true for $n=k+2$. As the result has been shown to be true for $n=1$ and $n=2$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 **cso**
			(5)
			10

	Question 8 Notes	
8. (i)	**Note**	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (i)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	"Assume for $n=k$, $\begin{pmatrix}5&-8\\2&-3\end{pmatrix}^k=\begin{pmatrix}1+4k&-8k\\2k&1-4k\end{pmatrix}$" satisfies the requirement "true for $n=k$"
	Note	"For $n\in\mathbb{Z}^+$, $\begin{pmatrix}5&-8\\2&-3\end{pmatrix}^n=\begin{pmatrix}1+4n&-8n\\2n&1-4n\end{pmatrix}$" satisfies the requirement "true for all n"
	Note	Give B0 for stating LHS = RHS by itself with no reference to $\text{LHS} = \text{RHS} = \begin{pmatrix}5&-8\\2&-3\end{pmatrix}$
	Note	Allow for B1 for stating either, $n=1$, $\begin{pmatrix}5&-8\\2&-3\end{pmatrix}=\begin{pmatrix}5&-8\\2&-3\end{pmatrix}$ or $\begin{pmatrix}5&-8\\2&-3\end{pmatrix}=\begin{pmatrix}1+4&-8\\2&1-4\end{pmatrix}$
	Note	E.g. $\begin{pmatrix}1+4k&-8k\\2k&1-4k\end{pmatrix}\begin{pmatrix}5&-8\\2&-3\end{pmatrix}=\begin{pmatrix}1+4(k+1)&-8(k+1)\\2(k+1)&1-4(k+1)\end{pmatrix}$ with no intermediate working is M1 dM0 A0 A0
	Note	E.g. Writing any of • $\begin{pmatrix}1+4k&-8k\\2k&1-4k\end{pmatrix}\begin{pmatrix}5&-8\\2&-3\end{pmatrix}=\begin{pmatrix}5+20k-16k&-8-32k+24k\\10k+2-8k&-16k-3+12k\end{pmatrix}=\begin{pmatrix}1+4(k+1)&-8(k+1)\\2(k+1)&1-4(k+1)\end{pmatrix}$ • $\begin{pmatrix}1+4k&-8k\\2k&1-4k\end{pmatrix}\begin{pmatrix}5&-8\\2&-3\end{pmatrix}=\begin{pmatrix}5+4k&-8-8k\\2+2k&-4k-3\end{pmatrix}=\begin{pmatrix}1+4(k+1)&-8(k+1)\\2(k+1)&1-4(k+1)\end{pmatrix}$ is M1 dM1 A1
(ii)	**Note**	Ignore $u_3=8u_2-12u_1=8(40)-12(8)=224$ as part of their solution to (i)
	Note	Ignore $\{n=3,\}$ $u_2=6^3+2^3=224$ as part of their solution to (i)
	Note	Full marks in (i) can be obtained for an equivalent proof where $n=k\rightarrow n=k-1$; i.e. $k\equiv k-1$
	Note	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (ii)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	"Assume for $n=k$, $u_k=6^k+2^k$ and for $n=k+1$, $u_{k+1}=6^{k+1}+2^{k+1}$" satisfies the requirement "true for $n=k$ and $n=k+1$"
	Note	"For $n\in\mathbb{Z}^+$, $u_n=6^n+2^n$" satisfies the requirement "true for all n"
	Note	Writing $u_{k+2}=8(6^{k+1}+2^{k+1})-12(6^k+2^k)=6^{k+2}+2^{k+2}$ with no intermediate working is M1 A0 A0 A0
	Note	E.g. Writing either • $u_{k+2}=8(6^{k+1}+2^{k+1})-12(6^k+2^k)=48(6^k)+16(2^k)-12(6^k+2^k)=6^{k+2}+2^{k+2}$ • $u_{k+2}=8(6^{k+1}+2^{k+1})-12(6^k+2^k)=36(6^k)+4(2^k)=6^{k+2}+2^{k+2}$ • $u_{k+2}=8(6^{k+1}+2^{k+1})-12(6^k+2^k)=6^2(6^k)+2^2(2^k)=6^{k+2}+2^{k+2}$ • $u_{k+2}=8(6^{k+1}+2^{k+1})-12(6^k+2^k)=8(6^{k+1}+2^{k+1})-2(6^{k+1})-6(2^{k+1})=6^{k+2}+2^{k+2}$ • $u_{k+2}=8(6^{k+1}+2^{k+1})-12(6^k+2^k)=6(6^{k+1})+2(2^{k+1})=6^{k+2}+2^{k+2}$ • $u_{k+2}=8(6^{k+1}+2^{k+1})-12(6^k+2^k)=8(6^{k+1})-2(6^{k+1})+4(2^{k+2})-3(2^{k+2})=6^{k+2}+2^{k+2}$ • $u_{k+2}=8(6^{k+1}+2^{k+1})-12(6^k+2^k)=(6)(6^{k+1})+4(2^{k+2})-3(2^{k+2})=6^{k+2}+2^{k+2}$ is M1 A1 A1
	Note	Writing $u_{k+2}=8(6^{k+1}+2^{k+1})-12(6^k+2^k)=(6)6^{k+1}+2^{k+2}=6^{k+2}+2^{k+2}$ with no intermediate working is M1 A0 A0 A0

	Question 8 Notes Continued	
8. (ii)	**Note**	Full marks in (i) can be obtained for an equivalent proof where e.g. • $n = k,\ n = k+1,\ \rightarrow n = k-2,\ n = k-1$; i.e. $k \equiv k-2$
8. (i), (ii)	**Note**	Referring to n as a real number their conclusion is final A0
	Note	Referring to n as any integer in their conclusion is final A0
	Note	Condone $n \in \mathbb{Z}^*$ as part of their conclusion for the final A1