Question Number	Scheme	Notes	Marks
9.	$z_1 = -1 - \mathrm{i},\ z_2 = 3 - 4\mathrm{i}$; (d) $\dfrac{p + \mathrm{i}q - 8z_1}{p - \mathrm{i}q - 8z_2} = 3\mathrm{i}$		
(a)	$z_1 - z_2 = -4 + 3\mathrm{i}$	$z_1 - z_2 = -4 + 3\mathrm{i}$, seen or implied	B1
	$\{z_1 - z_2 = -4 + 3\mathrm{i} \Rightarrow\}$ $\arg(z_1 - z_2) = \pi - \tan^{-1}\left(\frac{3}{4}\right)$ $\{\arg(z_1 - z_2) = \pi - 0.6435011... \Rightarrow\}$	$z_1 - z_2 = \alpha + \beta\mathrm{i};\ \alpha < 0,\ \beta > 0$ **and** uses trigonometry to find an expression for $\arg(z_1 - z_2)$ so that $\arg(z_1 - z_2)$ is in the range (1.58..., 3.14...) or (90°, 180°) or (−3.15..., −4.71...) or (−180°, −270°)	M1
	$\arg(z_1 - z_2) = 2.4980915...$ {= 2.498 (3 dp)}	awrt 2.498	A1
			(3)
(b) **Way 1**	$\left\{\dfrac{z_1}{z_2} =\right\}\ \dfrac{(-1-\mathrm{i})(3+4\mathrm{i})}{(3-4\mathrm{i})(3+4\mathrm{i})}$	Multiplies numerator and denominator by the conjugate of the denominator	M1
	$= \dfrac{-3-4\mathrm{i}-3\mathrm{i}+4}{9+16} \quad \left\{= \dfrac{1-7\mathrm{i}}{25}\right\}$	Numerator correct (with $\mathrm{i}^2 = -1$ applied) **or** denominator correct (with $\mathrm{i}^2 = -1$ applied)	A1
	$= \dfrac{1}{25} - \dfrac{7}{25}\mathrm{i}$ or $0.04 - 0.28\mathrm{i}$	$\dfrac{1}{25} - \dfrac{7}{25}\mathrm{i}$ or $0.04 - 0.28\mathrm{i}$	A1
			(3)
(b) **Way 2**	$\dfrac{-1-\mathrm{i}}{3-4\mathrm{i}} = a + \mathrm{i}b \Rightarrow -1 - \mathrm{i} = (a + \mathrm{i}b)(3 - 4\mathrm{i})$ $\{\text{Real} \Rightarrow\}\quad -1 = 3a + 4b$ $\{\text{Imaginary} \Rightarrow\}\quad -1 = -4a + 3b$ $\Rightarrow a = ...$ or $b = ...$	Sets $\dfrac{z_1}{z_2} = a + \mathrm{i}b$, multiplies both sides by z_2, attempts to equate **both** the real part **and** the imaginary part of the resulting equation **and** solves to give at least one of $a = ...$ or $b = ...$	M1
	$a = \dfrac{1}{25}$ or 0.04, $b = -\dfrac{7}{25}$ or -0.28	At least one of either a or b is correct	A1
	So, $\dfrac{z_1}{z_2} = \dfrac{1}{25} - \dfrac{7}{25}\mathrm{i}$ or $0.04 - 0.28\mathrm{i}$	$\dfrac{1}{25} - \dfrac{7}{25}\mathrm{i}$ or $0.04 - 0.28\mathrm{i}$	A1
			(3)
(c)	$\left\{\left\|\dfrac{z_1}{z_2}\right| =\right\}\ \sqrt{\left(\dfrac{1}{25}\right)^2 + \left(\dfrac{-7}{25}\right)^2} \quad \left\{\textbf{or}\ \dfrac{\|z_1\|}{\|z_2\|} =\right\}\ \dfrac{\sqrt{(-1)^2 + (-1)^2}}{\sqrt{(3)^2 + (-4)^2}}$	Finds $\left\|\dfrac{z_1}{z_2}\right|$ by applying a full Pythagoras method	M1
	$\left\{= \dfrac{\sqrt{50}}{25}\right\} = \dfrac{\sqrt{2}}{5}$	$\dfrac{\sqrt{2}}{5}$ or $\dfrac{1}{5}\sqrt{2}$	A1 **cao**
			(2)
(d)	$p + \mathrm{i}q - 8z_1 = 3\mathrm{i}(p - \mathrm{i}q - 8z_2)$ $\Rightarrow p + \mathrm{i}q - 8(-1 - \mathrm{i}) = 3\mathrm{i}(p - \mathrm{i}q - 8(3 - 4\mathrm{i}))$	Multiplies both sides by only $(p - \mathrm{i}q - 8z_2)$, and substitutes the given values for z_1 and z_2	M1
	$\Rightarrow p + \mathrm{i}q + 8 + 8\mathrm{i} = 3p\mathrm{i} + 3q - 72\mathrm{i} - 96$ $\{\text{Real} \Rightarrow\}\quad p + 8 = 3q - 96$ $\{\text{Imaginary} \Rightarrow\}\quad q + 8 = 3p - 72$	**dependent on the previous M mark** attempts to equate **both** the real part **and** the imaginary part of the resulting equation	dM1
		Both correct equations which can be simplified or un-simplified	A1
	$\begin{cases} p - 3q = -104 \\ 3p - q = 80 \end{cases} \Rightarrow \begin{cases} p - 3q = -104 \\ 9p - 3q = 240 \end{cases}$ $\Rightarrow p = 43,\ q = 49$	**dependent on the previous M mark** Obtains two equations both in terms of p and q and solves them simultaneously to give at least one of $p = ...$ or $q = ...$	ddM1
		$p = 43,\ q = 49$	A1
			(5)
			13

	Question 9 Notes	
9. (a)	**Note**	Allow M1 (implied) for awrt 2.5, truncated 2.4, awrt -3.8, truncated -3.7, awrt $143°$, awrt $-217°$ or truncated $-216°$
	Note	Give B1 M1 A1 for writing $\arg(z_1 - z_2) =$ awrt 2.498 from no working.
(b)	**Note**	Give 2nd A0 for writing down $\frac{1-7\text{i}}{25}$ with no reference to $\frac{1}{25} - \frac{7}{25}\text{i}$ or $0.04 - 0.28\text{i}$
	Note	Give M1 1st A1 for writing down $\frac{1-7\text{i}}{25}$ from no working in (b)
	Note	Give M1 A1 A1 for writing down $\frac{1-7\text{i}}{25} = \frac{1}{25} - \frac{7}{25}\text{i}$ or $0.04 - 0.28\text{i}$ from no working in (b)
	Note	Give M1 A1 A1 for writing down $\frac{1}{25} - \frac{7}{25}\text{i}$ or $0.04 - 0.28\text{i}$ from no working in (b)
	Note	Give 2nd A0 for simplifying a correct $\frac{1}{25} - \frac{7}{25}\text{i}$ to give a final answer of $1 - 7\text{i}$
(c)	**Note**	M1 can be implied by awrt 0.283 or truncated 0.282
	Note	Give A0 for $\frac{\sqrt{50}}{25}$ or 0.28284… without reference to $\frac{\sqrt{2}}{5}$ or $\frac{1}{5}\sqrt{2}$
	Note	Give M0 for $\sqrt{\left(\frac{1}{25}\right)^2 + \left(\frac{-7\text{i}}{25}\right)^2}$ unless recovered by later working
	Note	Give M1 A1 for writing $\left\|\frac{z_1}{z_2}\right\| = \frac{\sqrt{2}}{5}$ from no working.

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom