Summer 2019
WFM01/01 Further Pure Mathematics F1
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	$f(x)=5+4x^2-\frac{4}{3}x^3-\frac{7}{2x}\ ;\ x>0$		
(a)	$f'(x)=8x-4x^2+\frac{7}{2}x^{-2}$	At least one of either $5+4x^2\to\pm Ax$ or $-\frac{4}{3}x^3\to\pm Bx^2$ or $-\frac{7}{2x}\to\pm Cx^{-2}$; $A, B, C\neq 0$	M1
		Correct differentiation, which can be un-simplified or simplified	A1
			(2)
(b)	$f(0.5)=-\frac{7}{6},\ f'(0.5)=17$	Either $f(0.5)=-\frac{7}{6}$ or awrt -1.17 or truncated -1.16 **or** $f'(0.5)=17$ **or a correct numerical expression** for either $f(0.5)$ or $f'(0.5)$ **Can be implied by later working**	B1
	$\left\{\alpha\simeq 0.5-\frac{f(0.5)}{f'(0.5)}\right\}\Rightarrow\alpha\simeq 0.5-\frac{-\frac{7}{6}}{17}$	Valid attempt at Newton-Raphson using their values of $f(0.5)$ and $f'(0.5)$	M1
	$\left\{\alpha=0.56862745...\text{ or }\frac{29}{51}\right\}\Rightarrow\alpha=0.569$ (3 dp)	Correct $f'(x)$ and 0.569 on first iteration (Ignore any subsequent iterations)	A1 **cso** **cao**
	Correct differentiation followed by 0.569 (with no working seen) scores full marks in part (b)		**(3)**
(c) **Way 1**	$f(3)=\frac{23}{6}=3.833333...$ $f(3.5)=-\frac{25}{6}=-4.166666...$	Attempts to evaluate both $f(3)$ and $f(3.5)$ **and** either $f(3)=\frac{23}{6}$ or awrt 4 or truncated 3 **or** $f(3.5)=-\frac{25}{6}$ or awrt -4	M1
	Sign change {positive, negative} {and $f(x)$ is continuous} therefore a root $\{\beta\}$ exists in the interval {[3, 3.5]}	Both values correct awrt (or truncated) to 1sf, sign change and conclusion.	A1
			(2)
(d) **Way 1**	$\frac{\beta-3}{"3.8333..."}=\frac{3.5-\beta}{"4.1666..."}$ **or** $\frac{\beta-3}{3.5-\beta}=\frac{"3.8333..."}{"4.1666..."}$ **or** $\frac{\beta-3}{"3.8333..."}=\frac{3.5-3}{"4.166..."+"3.8333..."}$	A correct linear interpolation method. Do not allow this mark if a total of one or three negative lengths are used or if either fraction is the wrong way up. This mark may be implied.	M1
	• $\beta=\left(\frac{(3)("4.1666...")+(3.5)("3.8333...")}{"4.1666..."+"3.8333..."}\right)=\left(\frac{12.5+13.4166...}{8}\right)$ • $\beta=3+\left(\frac{"3.8333..."}{"4.1666..."+"3.8333..."}\right)(0.5)$ or $\beta=3+\left(\frac{"\frac{23}{6}"}{"8"}\right)(0.5)$ • $\beta=3+\left(\frac{"-3.8333..."}{"-4.1666..."+"-3.8333..."}\right)(0.5)$	**dependent on the previous M mark** Rearranges to give $\beta=...$	dM1
	$\left\{\beta=3.239583...\text{ or }3\frac{23}{96}\text{ or }\frac{311}{96}\right\}\Rightarrow\beta=3.24$ (2dp)	awrt 3.24 (Ignore any subsequent iterations)	A1
			(3)
			10

Question Number	Scheme	Notes	Marks
1. (d) **Way 2**	$\frac{x}{"3.8333..."} = \frac{0.5-x}{"4.1666..."}$ $x = \frac{(0.5)("3.8333...")}{"3.8333..." + "4.1666..."} = 0.239583...$ $\Rightarrow \beta = 3 + 0.239583...$	Finds x using a correct method of similar triangles and applies "3 + their x"	M1 dM1
	$\left\{\beta = 3.239583... \text{ or } 3\frac{23}{96} \text{ or } \frac{311}{96}\right\} \Rightarrow \beta = 3.24 \ (2\text{dp})$	awrt 3.24	A1
			(3)
1. (d) **Way 3**	$\frac{0.5-x}{"3.8333..."} = \frac{x}{"4.1666..."}$ $x = \frac{(0.5)("4.1666...")}{"3.8333..." + "4.1666..."} = 0.260416...$ $\Rightarrow \beta = 3.5 - 0.260416...$	Finds x using a correct method of similar triangles and applies "3.5 – their x"	M1 dM1
	$\left\{\beta = 3.239583... \text{ or } 3\frac{23}{96} \text{ or } \frac{311}{96}\right\} \Rightarrow \beta = 3.24 \ (2\text{dp})$	awrt 3.24	A1
			(3)

	Question 1 Notes	
1. (b)	**Note**	Give full marks in part (b) for correct differentiation in (a) followed by the correct answer in (b) with no working.
	M1	This mark can be implied by applying at least one correct ***value*** of either $f(0.5)$ or their $f'(0.5)$ (where $f'(0.5)$ is found using their $f'(x)$) to 1 significant figure in $0.5 - \frac{f(0.5)}{f'(0.5)}$. So ***just writing*** $0.5 - \frac{f(0.5)}{f'(0.5)}$ with an incorrect ft answer on their $f'(0.5)$ scores B0M0A0.
	Note	Give B1M1A0 for a correct $f'(x)$ in (a) followed by only $\alpha \simeq 0.5 - \frac{f(0.5)}{f'(0.5)} = \frac{29}{51}$ in (b)
	Note	**Differentiating INCORRECTLY to give** $f'(x) = 8x - 4x^2 + 14x^{-2}$ leads to $\alpha \simeq 0.5 - \frac{-\frac{7}{6}}{59} = \frac{92}{177} = 0.5197740113... = 0.520$ (3 dp) **This response should be given B1 M1 A0**
	Note	**Differentiating INCORRECTLY to give** $f'(x) = 8x - 4x^2 + 14x^{-2}$ and $\alpha \simeq 0.5 - \frac{f(0.5)}{f'(0.5)} = 0.520$ or truncated 0.52 or 0.519 or awrt 0.520 is B1 M1 A0
(c)	**Note**	**Way 1: correct solution only** Required to state **both** values for $f(3)$ and $f(3.5)$ correct awrt (or truncated) to 1sf along with **a reason and a conclusion.** Reference to change of sign **or** e.g. $f(3) \times f(3.5) < 0$ **or** $f(3) > 0 > f(3.5)$ **or** a diagram **or** < 0 and > 0 **or** one positive, one negative are sufficient reasons. There must be a conclusion, e.g. {x or} $\beta \in [3, 3.5]$ or {x or} $\beta \in (3, 3.5)$ or root lies between 3 and 3.5. Ignore the presence or absence of any reference to continuity.
	Note	A minimal acceptable reason and conclusion is "change of sign, so $\beta \in [3, 3.5]$" **or** "change of sign, so root is between 3 and 3.5" **or** "change of sign, so root"

	Question 1 Notes Continued	
1. (c)	**Note**	**<u>Way 2</u>** The root of $f(x)=0$ is $3.27491258...$, so they can choose x_1 which is less than $3.27491258...$ and choose x_2 which is greater than $3.27491258...$ with both x_1 and x_2 lying in the interval [3, 3.5]. **M1:** Finds $f(x_1)$ and $f(x_2)$ with one of these values correct awrt (or truncated) to 1sf **A1:** Both values correct awrt (or truncated) to 1sf, sign change and conclusion.
	Note	**<u>Helpful Table</u>** x | $f(x)$ 3 | 3.83333333... 3.1 | 2.58963440... 3.2 | 1.17558333... 3.3 | −0.41660606... 3.4 | −2.19474509... 3.5 | −4.16666666...
1. (d)	**Note**	Condone writing the symbol α in place of β in part (d)
	Note	$\dfrac{\beta-3}{3.5-\beta}=\left\lvert\dfrac{"3.833..."}{"-4.1666..."}\right\rvert$ is a valid method for the first M mark
	Note	Give 1st M1 for either $\dfrac{f(3)}{-f(3.5)}=\dfrac{\beta-3}{3.5-\beta}$ or $\dfrac{f(1.2)}{\lvert f(1.3)\rvert}=\dfrac{\beta-3}{3.5-\beta}$ or $\dfrac{\lvert f(3)\rvert}{\lvert f(3.5)\rvert}=\dfrac{\beta-3}{3.5-\beta}$
	Note	Give M1 dM1 A1 for the correct statement $\dfrac{3\lvert f(3.5)\rvert+3.5f(3)}{\lvert f(3.5)\rvert+f(3)}=3.24$
	Note	Give M0 dM0 for $\dfrac{3\lvert f(3.5)\rvert+3.5f(3)}{\lvert f(3.5)\rvert+f(3)}=\dfrac{3("-4.166...")+3.5("3.8333...")}{("-4.166...")+("3.8333...")}$
	Note	Give M1 dM1 for the correct statement $\beta=\dfrac{3.5+3k}{k+1}$, where k is defined as $k=\dfrac{\lvert f(3.5)\rvert}{f(3)}=\dfrac{4.1666...}{3.8333...}=1.086957...$
	Note	Give M1 dM1 for the correct statement $\beta=\dfrac{3+3.5c}{c+1}$, where c is defined as $c=\dfrac{f(3)}{\lvert f(3.5)\rvert}=\dfrac{3.8333...}{4.1666...}=0.92$
	Note	$\dfrac{\beta-3}{3.5-\beta}=\dfrac{"3.8333..."}{"4.1666..."}\Rightarrow\beta=3.24$ with no intermediate working is M1 dM1 A1
	Note	$\dfrac{\beta-3}{3.8333...}=\dfrac{3.5-\beta}{-4.1666...}\Rightarrow\beta=-2.75$ is M0 dM0 A0
	Note	$\dfrac{\beta-3}{-3.8333...}=\dfrac{3.5-\beta}{-4.1666...}\Rightarrow\beta=3.24$ is M1 dM1 A1
	Note	$\dfrac{\beta-3}{3.5-\beta}=\dfrac{"4.1666..."}{"3.8333..."}\Rightarrow\beta=3.260416...$ is M0 dM0 A0

Question Number	Scheme	Notes	Marks
1. (d) **Way 4**	• $y-\frac{23}{6}=\frac{-\frac{25}{6}-\frac{23}{6}}{3.5-3}(x-3) \Rightarrow 0-\frac{23}{6}=\frac{-\frac{25}{6}-\frac{23}{6}}{3.5-3}(x-3)$ • $y--\frac{25}{6}=\frac{-\frac{25}{6}-\frac{23}{6}}{3.5-3}(x-3.5) \Rightarrow 0--\frac{25}{6}=\frac{-\frac{25}{6}-\frac{23}{6}}{3.5-3}(x-3.5)$	Complete method of finding a line joining the points $(3, f(3))$, $(3.5, f(3.5))$ followed by setting $y=0$	M1
	$\Rightarrow x=...$ or $\beta=...$	**dependent on the previous M mark** Rearranges to give $x=...$ or $\beta=...$	dM1
	$\left\{x \text{ or } \beta=3.239583... \text{ or } 3\frac{23}{96} \text{ or } \frac{311}{96}\right\} \Rightarrow \beta=3.24 \ (2\text{dp})$	awrt 3.24	A1
			(3)