Question Number	Scheme	Notes	Marks
6.	$2x^2+x+4=0$ has roots α, β		
(a)	$\alpha+\beta=-\frac{1}{2},\ \alpha\beta=2$	**Both** $\alpha+\beta=-\frac{1}{2}$ **and** $\alpha\beta=2$	B1
			(1)
(b)(i)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=...$	**Use** of a **correct** identity for $\alpha^2+\beta^2$ May be implied by their work	M1
	$=\left(-\frac{1}{2}\right)^2-2(2)=-\frac{15}{4}$	$-\frac{15}{4}$ or -3.75 or $-3\frac{3}{4}$ **from correct working**	A1 **cso**
(ii)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=...$ **or** $=(\alpha+\beta)((\alpha+\beta)^2-3\alpha\beta)=...$ **or** $=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)=...$	**Use** of a **correct** identity for $\alpha^3+\beta^3$ May be implied by their work	M1
	$=\left(-\frac{1}{2}\right)^3-3(2)\left(-\frac{1}{2}\right)=\frac{23}{8}$ **or** $=\left(-\frac{1}{2}\right)\left(\left(-\frac{1}{2}\right)^2-3(2)\right)=\frac{23}{8}$ **or** $=\left(-\frac{1}{2}\right)\left(-\frac{15}{4}-2\right)=\frac{23}{8}$	$\frac{23}{8}$ or 2.875 or $2\frac{7}{8}$ **from correct working**	A1 **cso**
			(4)
(c)	$\sum=\alpha^3+\frac{1}{\beta}+\beta^3+\frac{1}{\alpha}$ $=\alpha^3+\beta^3+\frac{\alpha+\beta}{\alpha\beta}$	$\sum=\frac{\alpha^3\beta+1}{\beta}+\frac{\alpha\beta^3+1}{\alpha}$ $=\frac{\alpha\beta(\alpha^3+\beta^3)+(\alpha+\beta)}{\alpha\beta}$	Simplifies $\frac{1}{\beta}+\frac{1}{\alpha}$ to give $\frac{\alpha+\beta}{\alpha\beta}$ **(can be implied)** **and** uses at least two of their $\alpha^3+\beta^3$, $\alpha+\beta$ or $\alpha\beta$ in an attempt to find a **numerical value** for the sum of $\left(\alpha^3+\frac{1}{\beta}\right)$ and $\left(\beta^3+\frac{1}{\alpha}\right)$	M1
	e.g. $=\frac{23}{8}+\frac{\left(-\frac{1}{2}\right)}{2}=\frac{21}{8}$ **or** $=\frac{2(\frac{23}{8})+(-\frac{1}{2})}{2}$			
	$\Pi=\left(\alpha^3+\frac{1}{\beta}\right)\left(\beta^3+\frac{1}{\alpha}\right)$ $=(\alpha\beta)^3+\alpha^2+\beta^2+\frac{1}{\alpha\beta}$	$\Pi=\left(\frac{\alpha^3\beta+1}{\beta}\right)\left(\frac{\alpha\beta^3+1}{\alpha}\right)$ $=\frac{\alpha^4\beta^4+\alpha^3\beta+\alpha\beta^3+1}{\alpha\beta}$ $=\frac{(\alpha\beta)^4+\alpha\beta(\alpha^2+\beta^2)+1}{\alpha\beta}$	Expands $\left(\alpha^3+\frac{1}{\beta}\right)\left(\beta^3+\frac{1}{\alpha}\right)$ to give 4 terms and uses at least one of their $\alpha\beta$ or $\alpha^2+\beta^2$ in an attempt to find a **numerical value** for the product	M1
	e.g. $=(2)^3+\left(-\frac{15}{4}\right)+\frac{1}{2}=\frac{19}{4}$ **or** $=\frac{(2)^4+2(-\frac{15}{4})+1}{2}$			
	$x^2-\frac{21}{8}x+\frac{19}{4}=0$	**Applies** $x^2-(\text{sum})x+\text{product}$ (can be implied), for their numerical values of the sum and product. **Note:** "$=0$" is not required for this mark		M1
	$8x^2-21x+38=0$	***Any integer multiple*** of $8x^2-21x+38=0$, including the "$=0$"		A1 **cso**
				(4)
				9

	Question 6 Notes	
6. (b)(i)	**Note**	Writing a correct $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ without attempting to substitute at least one of either their $\alpha+\beta$ or their $\alpha\beta$ into $(\alpha+\beta)^2-2\alpha\beta$ is M0
	Note	An incorrect $\alpha+\beta=\frac{1}{2}, \alpha\beta=2$ from (a) leading to $\alpha^2+\beta^2=\left(\frac{1}{2}\right)^2-2(2)=-\frac{15}{4}$ is M1 A0
	Note	Give M1 A1 for writing down $\alpha^2+\beta^2=-\frac{15}{4}$, if they give $\alpha+\beta=-\frac{1}{2}, \alpha\beta=2$ in (a)
(b)(ii)	**Note**	Allow M1 A1 for $\alpha^3+\beta^3=(\alpha^2+\beta^2)(\alpha+\beta)-\alpha\beta(\alpha+\beta)=\left(-\frac{15}{4}\right)\left(-\frac{1}{2}\right)-(2)\left(-\frac{1}{2}\right)=\frac{23}{8}$
	Note	E.g. writing a correct $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ ***without*** attempting to substitute at least one of either their $\alpha+\beta$ or their $\alpha\beta$ into $(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ is M0
	Note	E.g. writing a correct $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ ***without*** attempting to substitute at least one of either their $\alpha+\beta$, their $\alpha^2+\beta^2$ or their $\alpha\beta$ into $(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ is M0
	Note	Give M1 A1 for writing down $\alpha^3+\beta^3=\frac{23}{8}$, if they give $\alpha+\beta=-\frac{1}{2}, \alpha\beta=2$ in (a)
(b)	**ALT**	They can use the equation $2x^2+x+4=0$ with roots α, β to give $\left.\begin{matrix}2\alpha^2+\alpha+4=0\\2\beta^2+\beta+4=0\end{matrix}\right\} \Rightarrow 2\alpha^2+2\beta^2+\alpha+\beta+8=0$ So, $\alpha^2+\beta^2=\frac{1}{2}(-(\alpha+\beta)-8))=\frac{1}{2}\left(--\frac{1}{2}-8\right)=\frac{1}{2}\left(\frac{1}{2}-8\right)=-\frac{15}{4}$ $\left.\begin{matrix}2\alpha^3+\alpha^2+4\alpha=0\\2\beta^3+\beta^2+4\beta=0\end{matrix}\right\} \Rightarrow 2\alpha^3+2\beta^3+\alpha^2+\beta^2+4\alpha+4\beta=0$ So, $\alpha^3+\beta^3=\frac{1}{2}(-(\alpha^2+\beta^2)-4(\alpha+\beta)))=\frac{1}{2}\left(--\frac{15}{4}-4\left(-\frac{1}{2}\right)\right)=\frac{1}{2}\left(\frac{15}{4}+2\right)=\frac{23}{8}$
(a)	**Note**	Give B0 for α, $\beta=\frac{-1+\sqrt{31}\mathrm{i}}{4}, \frac{-1-\sqrt{31}\mathrm{i}}{4}$ and then stating that $\alpha+\beta=-\frac{1}{2}$, $\alpha\beta=2$
	Note	Give B0 for $\alpha+\beta=\frac{-1+\sqrt{31}\mathrm{i}}{4}+\frac{-1-\sqrt{31}\mathrm{i}}{4}=-\frac{1}{2}$ and $\alpha\beta=\left(\frac{-1+\sqrt{31}\mathrm{i}}{4}\right)\left(\frac{-1-\sqrt{31}\mathrm{i}}{4}\right)=2$
(b)(i)	**Note**	Give M0 A0 for $\alpha^2+\beta^2=\left(\frac{-1+\sqrt{31}\mathrm{i}}{4}\right)^2+\left(\frac{-1-\sqrt{31}\mathrm{i}}{4}\right)^2=-\frac{15}{4}$
(b)(ii)	**Note**	Give M0 A0 for $\alpha^3+\beta^3=\left(\frac{-1+\sqrt{31}\mathrm{i}}{4}\right)^3+\left(\frac{-1-\sqrt{31}\mathrm{i}}{4}\right)^3=\frac{23}{8}$
(b)	**Note**	Using $\frac{-1+\sqrt{31}\mathrm{i}}{4}, \frac{-1-\sqrt{31}\mathrm{i}}{4}$ to find $\alpha+\beta=-\frac{1}{2}$, $\alpha\beta=2$ followed by • $\alpha^2+\beta^2==\left(-\frac{1}{2}\right)^2-2(2)=-\frac{15}{4}$, scores M1 A0 in (b)(i) • e.g. $\alpha^3+\beta^3=\left(-\frac{1}{2}\right)^3-3(2)\left(-\frac{1}{2}\right)=\frac{23}{8}$, scores M1 A1 in (b)(ii)
(c)	**Note**	A correct method leading to $p=8$, $q=-21$, $r=38$ without writing a final answer of $8x^2-21x+38=0$ is final M1 A0

	Question 6 Notes Continued	
6. (c)	**Note**	Using $\frac{-1+\sqrt{31}\text{i}}{4}, \frac{-1-\sqrt{31}\text{i}}{4}$ explicitly to find the sum and product of $\alpha^3+\frac{1}{\beta}$ and $\beta^3+\frac{1}{\alpha}$ • i.e. sum $= \left(\frac{-1+\sqrt{31}\text{i}}{4}\right)^3+\frac{1}{\left(\frac{-1-\sqrt{31}\text{i}}{4}\right)}+\left(\frac{-1-\sqrt{31}\text{i}}{4}\right)^3+\frac{1}{\left(\frac{-1+\sqrt{31}\text{i}}{4}\right)}=\frac{21}{8}$ • ie. product $=\left(\left(\frac{-1+\sqrt{31}\text{i}}{4}\right)^3+\frac{1}{\left(\frac{-1-\sqrt{31}\text{i}}{4}\right)}\right)\left(\left(\frac{-1-\sqrt{31}\text{i}}{4}\right)^3+\frac{1}{\left(\frac{-1+\sqrt{31}\text{i}}{4}\right)}\right)=\frac{19}{4}$ • $x^2-\frac{21}{8}x+\frac{19}{4}=0 \Rightarrow 8x^2-21x+38=0$ scores M0 M0 M1 A0 in part (c).
	Note	Using $\frac{-1+\sqrt{31}\text{i}}{4}, \frac{-1-\sqrt{31}\text{i}}{4}$ to find $\alpha+\beta=-\frac{1}{2},\ \alpha\beta=2$ **and applying** $\alpha+\beta=-\frac{1}{2},\ \alpha\beta=2$ can potentially score full marks in (c). E.g. • sum $= \alpha^3+\beta^3+\frac{\alpha+\beta}{\alpha\beta}=\frac{23}{8}+\frac{\left(-\frac{1}{2}\right)}{2}=\frac{21}{8}$ • product $=(\alpha\beta)^3+\alpha^2+\beta^2+\frac{1}{\alpha\beta}=(2)^3+\left(-\frac{15}{4}\right)+\frac{1}{2}=\frac{19}{4}$ • $x^2-\frac{21}{8}x+\frac{19}{4}=0 \Rightarrow 8x^2-21x+38=0$
	Note	Give final M0 for $\sum=\frac{21}{8}$, $\mathbf{\Pi}=\frac{19}{4}$ leading to $x^2-\frac{21}{8}+\frac{19}{4}=0$ (without recovery)
	Note	Allow final M1 for $\sum=\frac{21}{8}$, $\mathbf{\Pi}=\frac{19}{4}$ with $x^2-(\text{sum})x+(\text{product})$ leading to $x^2-\frac{21}{8}+\frac{19}{4}=0$
	Note	An alternative method uses a correct $\left(x-\alpha^3-\frac{1}{\beta}\right)\left(x-\beta^3-\frac{1}{\alpha}\right)=0$
	Note	Allow 1st M1 and/or 2nd M1 for using an incorrect $\left(x-\alpha^3+\frac{1}{\beta}\right)\left(x-\beta^3+\frac{1}{\alpha}\right)=0$
	Note	Give final M0 for an incorrect $\left(x-\alpha^3+\frac{1}{\beta}\right)\left(x-\beta^3+\frac{1}{\alpha}\right)=0$ unless recovered
	Note	When expanding $\left(\alpha^3+\frac{1}{\beta}\right)\left(\beta^3+\frac{1}{\alpha}\right)$ to give $(\alpha\beta)^3+\alpha^2+\beta^2+\frac{1}{\alpha\beta}$, some will write $\frac{\alpha+\beta}{\alpha\beta}$ in place of $\frac{1}{\alpha\beta}$ So, allow 2nd M1 for expanding $\left(\alpha^3+\frac{1}{\beta}\right)\left(\beta^3+\frac{1}{\alpha}\right)$ to give $(\alpha\beta)^3+\alpha^2+\beta^2+\frac{\alpha+\beta}{\alpha\beta}$ and using at least one of their $\alpha\beta$ or $\alpha^2+\beta^2$ in an attempt to find a **numerical value** for the product.