Question Number	Scheme	Notes	Marks
8.	$f(n)=3^{4n-2}+2^{6n-3}$ is divisible by 17		
Way 1	$f(1)=3^2+2^3=17$ {is divisible by 17}	$f(1)=17$ is the minimum	B1
	$f(k+1)-f(k)=3^{4(k+1)-2}+2^{6(k+1)-3}-(3^{4k-2}+2^{6k-3})$	Attempts $f(k+1)-f(k)$	M1
	$f(k+1)-f(k)=80(3^{4k-2})+63(2^{6k-3})$		
	$=80(3^{4k-2}+2^{6k-3})-17(2^{6k-3})$	$80(3^{4k-2}+2^{6k-3})$ or $80f(k)$; $-17(2^{6k-3})$	A1; A1
	or $=63(3^{4k-2}+2^{6k-3})+17(3^{4k-2})$	$63(3^{4k-2}+2^{6k-3})$ or $63f(k)$; $+17(3^{4k-2})$	
	$f(k+1)=80(3^{4k-2}+2^{6k-3})-17(2^{6k-3})+f(k)$ **or** $f(k+1)=63(3^{4k-2}+2^{6k-3})+17(3^{4k-2})+f(k)$ **or** $f(k+1)=80f(k)-17(2^{6k-3})+f(k)$ **or** $f(k+1)=63f(k)+17(3^{4k-2})+f(k)$	**dependent on at least one of the previous A marks being gained** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2}+2^{6k-3})$	dM1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n $(\in\mathbb{Z}^+)$		A1 cso
			(6)
Way 2	$f(1)=3^2+2^3=17$ {is divisible by 17}	$f(1)=17$ is the minimum	B1
	$f(k+1)=3^{4(k+1)-2}+2^{6(k+1)-3}$	Attempts $f(k+1)$	M1
	$f(k+1)=81(3^{4k-2})+64(2^{6k-3})$		
	$=81(3^{4k-2}+2^{6k-3})-17(2^{6k-3})$	$81(3^{4k-2}+2^{6k-3})$ or $81f(k)$; $-17(2^{6k-3})$	A1; A1
	or $=64(3^{4k-2}+2^{6k-3})+17(3^{4k-2})$	$64(3^{4k-2}+2^{6k-3})$ or $64f(k)$; $+17(3^{4k-2})$	
	$f(k+1)=81(3^{4k-2}+2^{6k-3})-17(2^{6k-3})$ **or** $f(k+1)=64(3^{4k-2}+2^{6k-3})+17(3^{4k-2})$ **or** $f(k+1)=81f(k)-17(2^{6k-3})$ **or** $f(k+1)=64f(k)+17(3^{4k-2})$	**dependent on at least one of the previous A marks being gained** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2}+2^{6k-3})$	dM1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n $(\in\mathbb{Z}^+)$		A1 cso
			(6)
Way 3	**General Method:** Using $f(k+1)-mf(k), m\in\mathbb{Z}$		
	$f(1)=3^2+2^3=17$ {is divisible by 17}	$f(1)=17$ is the minimum	B1
	$f(k+1)-mf(k)=3^{4(k+1)-2}+2^{6(k+1)-3}-m(3^{4k-2}+2^{6k-3})$	Attempts $f(k+1)-mf(k)$	M1
	$f(k+1)-mf(k)=(81-m)(3^{4k-2})+(64-m)(2^{6k-3})$		
	$=(81-m)(3^{4k-2}+2^{6k-3})-17(2^{6k-3})$ or	$(81-m)(3^{4k-2}+2^{6k-3})$ or $(81-m)f(k)$; $-17(2^{6k-3})$	A1; A1
	$=(64-m)(3^{4k-2}+2^{6k-3})+17(3^{4k-2})$	$(64-m)(3^{4k-2}+2^{6k-3})$ or $(64-m)f(k)$; $+17(3^{4k-2})$	
	$f(k+1)=(81-m)(3^{4k-2}+2^{6k-3})-17(2^{6k-3})+mf(k)$ or $f(k+1)=(64-m)(3^{4k-2}+2^{6k-3})+17(3^{4k-2})+mf(k)$ or $f(k+1)=(81-m)f(k)-17(2^{6k-3})+mf(k)$ **or** $f(k+1)=(64-m)f(k)+17(3^{4k-2})+mf(k)$	**dependent on at least one of the previous A marks being gained** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2}+2^{6k-3})$	dM1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n $(\in\mathbb{Z}^+)$		A1 cso
			(6)
			6

Question Number	Scheme	Notes	Marks
8.	$f(n)=3^{4n-2}+2^{6n-3}$ is divisible by 17		
Way 4	**General Method:** Using $f(k+1)-mf(k), m\in\mathbb{Z}$		
	$f(1)=3^2+2^3=17$ {is divisible by 17}	$f(1)=17$ is the minimum	B1
	$f(k+1)-mf(k)=3^{4(k+1)-2}+2^{6(k+1)-3}-m(3^{4k-2}+2^{6k-3})$	Attempts $f(k+1)-mf(k)$	M1
	$f(k+1)-mf(k)=(81-m)(3^{4k-2})+(64-m)(2^{6k-3})$		
	E.g. $m=47\Rightarrow f(k+1)-47f(k)=34(3^{4k-2})+17(2^{6k-3})$	$m=47$ and $34(3^{4k-2})$	A1
		$m=47$ and $17(2^{6k-3})$	A1
	$f(k+1)=34(3^{4k-2})+17(2^{6k-3})+47f(k)$	**dependent on at least one of the previous A marks being gained** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$	dM1
	If the result is <u>true for $n=k$,</u> then it is <u>true for $n=k+1$.</u> As the result has been shown to be <u>true for $n=1$,</u> then the result <u>is true for all n</u> $(\in\mathbb{Z}^+)$		A1 cso
			(6)
	In Way 4 there are many alternatives. See below for examples of alternatives where $m=30$ and $m=13$		
	The A1A1dM1 marks for some alternatives using $f(k+1)-mf(k), m\in\mathbb{Z}$		
Way 4.1	$f(k+1)=81(3^{4k-2})+64(2^{6k-3})$		
	$=30(3^{4k-2})+30(2^{6k-3})+51(3^{4k-2})+34(2^{6k-3})$		
	$=30(3^{4k-2}+2^{6k-3})+51(3^{4k-2})+34(2^{6k-3})$	$m=30$ and $51(3^{4k-2})$	A1
		$m=30$ and $34(2^{6k-3})$	A1
	$f(k+1)=30(3^{4k-2}+2^{6k-3})+51(3^{4k-2})+34(2^{6k-3})$ **or** $f(k+1)=30f(k)+51(3^{4k-2})+34(2^{6k-3})$	**dependent on at least one of the previous A marks being gained** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2}+2^{6k-3})$	dM1
Way 4.2	$f(k+1)=81(3^{4k-2})+64(2^{6k-3})$		
	$=13(3^{4k-2})+13(2^{6k-3})+68(3^{4k-2})+51(2^{6k-3})$		
	$=13(3^{4k-2}+2^{6k-3})+68(3^{4k-2})+51(2^{6k-3})$	$m=13$ and $68(3^{4k-2})$	A1
		$m=13$ and $51(2^{6k-3})$	A1
	$f(k+1)=13(3^{4k-2}+2^{6k-3})+68(3^{4k-2})+51(2^{6k-3})$ **or** $f(k+1)=13f(k)+68(3^{4k-2})+51(2^{6k-3})$	**dependent on at least one of the previous A marks being gained** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2}+2^{6k-3})$	dM1

	Question 8 Notes				
	Note	$f(n)=3^{4n-2}+2^{6n-3}$ can be written as $f(n)=3^{4n-2}+8^{2n-1}$			
Way 5	$f(n)=3^{4n-2}+2^{6n-3}=3^{4n-2}+8^{2n-1}$				
	$f(1)=3^2+8^1=17$ {is divisible by 17}		$f(1)=17$ is the minimum		B1
	$f(k+1)-f(k)=3^{4(k+1)-2}+8^{2(k+1)-1}-(3^{4k-2}+8^{2k-1})$		Attempts $f(k+1)-f(k)$		M1
	$f(k+1)-f(k)=80(3^{4k-2})+63(8^{2k-1})$				
	$=80(3^{4k-2}+8^{2k-1})-17(8^{2k-1})$ **or** $=63(3^{4k-2}+8^{2k-1})+17(3^{4k-2})$	$80(3^{4k-2}+8^{2k-1})$ or $80f(k)$; $63(3^{4k-2}+8^{2k-1})$ or $63f(k)$;	$-17(8^{2k-1})$ $+17(3^{4k-2})$	A1;	A1
	$f(k+1)=80(3^{4k-2}+8^{2k-1})-17(8^{2k-1})+f(k)$ **or** $f(k+1)=63(3^{4k-2}+8^{2k-1})+17(3^{4k-2})+f(k)$ **or** $f(k+1)=80f(k)-17(8^{2k-1})+f(k)$ **or** $f(k+1)=63f(k)+17(3^{4k-2})+f(k)$	**dependent on at least one of the previous A marks being gained** Makes $f(k+1)$ the subject and expresses it in terms of $f(k)$ and/or $(3^{4k-2}+8^{2k-1})$			dM1
	If the result is true for $n=k$, then it is true for $n=k+1$. As the result has been shown to be true for $n=1$, then the result is true for all n $(\in\mathbb{Z}^+)$				A1 cso
					(6)

	Note	Some students may set $f(k)=17M$ and so may prove the following general results • $\{f(k+1)=81f(k)-17(2^{6k-3})\}\Rightarrow f(k+1)=1377M-17(2^{6k-3})$ or $=17(3^4M-2^{6k-3})$ • $\{f(k+1)=64f(k)+17(3^{4k-2})\}\Rightarrow f(k+1)=1088M+17(3^{4k-2})$ or $=17(2^6M+3^{4k-2})$
	Note	**Final A1 mark is dependent on all previous marks being scored in Q8**
	Note	**Final A1:** There must be a correct final expression for $f(k+1)$ **and** a correct conclusion. The conclusion must convey the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.
	Note	Allow as part of their conclusion "true for all positive values of n"
	Note	Allow as part of their conclusion "true for all values of n"
	Note	Allow as part of their conclusion "true for all $n\in\mathbb{N}$"
	Note	Referring to n as a real number in their conclusion (e.g. true for all $n\in\mathbb{R}$) is final A0
	Note	Condone $n\in\mathbb{Z}^*$ as part of their conclusion for the final A1 mark
	Note	Allow $f(k+1)=3^4f(k)-17(2^{6k-3})$ as a correct alternative to $f(k+1)=81f(k)-17(2^{6k-3})$
	Note	Allow $f(k+1)=2^6f(k)+17(3^{4k-2})$ as a correct alternative to $f(k+1)=64f(k)+17(3^{4k-2})$