Question Number	Scheme	Notes	Marks
2.	Let $f(x)=3x^3+kx^2+33x+13$; $k\in\mathbb{R}$; $x=-\frac{1}{3}$ is a root of $f(x)=0$		
	Note: Ignore labelling of parts when marking Q2		
(a) **Way 1**	$3\left(-\frac{1}{3}\right)^3+k\left(-\frac{1}{3}\right)^2+33\left(-\frac{1}{3}\right)+13=0 \Rightarrow k=...$	Some evidence of substituting $x=-\frac{1}{3}$ into the given equation and solves to find $k=...$	M1
	$\left\{-\frac{1}{9}+\frac{1}{9}k-11+13=0\Rightarrow -1+k+18=0\Rightarrow\right\} k=-17$	$k=-17$	A1
			(2)
(a) **Way 2**	$f(x)=(3x+1)(x^2+Ax+13)$ x: $3(13)+A=33\Rightarrow A=-6$ x^2: $k=1+("-6")(3)$	Expresses $f(x)=(3x\pm1)(x^2+Ax\pm13)$, equates x terms to find A and equates x^2 terms to find k	M1
	$k=-17$	$k=-17$	A1
			(2)
(b)	$\{f(x)=\}\ (3x+1)(x^2-6x+13)$ or $\{f(x)=\}\ \left(x+\frac{1}{3}\right)(3x^2-18x+39)$	Attempts to find the quadratic factor e.g. using long division to obtain $(3x\pm1)$ with $(x^2\pm qx+...)$ or $\left(x\pm\frac{1}{3}\right)$ with $(3x^2\pm qx+...)$; $q=$ value $\neq 0$ e.g. factorising/equating coefficients to obtain $f(x)=(3x\pm1)(x^2\pm qx\pm r)$ or $f(x)=\left(x\pm\frac{1}{3}\right)(3x^2\pm qx\pm r)$, $q=$ value $\neq 0$, r can be 0	M1
		$x^2-6x+13$ or $3x^2-18x+39$ seen in their working	A1
	$\{x^2-6x+13=0$ or $3x^2-18x+39=0\Rightarrow\}$		
	e.g. • $x=\frac{--6\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)}$ • $(x-3)^2-9+13=0\Rightarrow x=...$	**dependent on the previous M mark** Correct method of applying the quadratic formula or completing the square **for solving their 3TQ** on their quadratic factor	dM1
	$\{x=\}\ 3\pm2i$ (or $3\pm i2$)	$3+2i$ **and** $3-2i$	A1
			(4)
			6

	Question 2 Notes	
2. (b)	**Note**	You can assume $z\equiv x$ for solutions in this part
	Note	Give final dM1A1 for $x^2-6x+13=0$ or $3x^2-18x+39=0\Rightarrow x=3+2i, 3-2i$ with no intermediate working.
	Note	Give M1 A1 dM1 A1 for $3x^3-17x^2+33x+13=0\Rightarrow x=3+2i, 3-2i$ with no intermediate working.
	Note	They must be solving a 3TQ $"A"x^2+"B"x+"C"$ where A, B, C ***are all numerical values*** $\neq 0$ for the final dM1 mark.
	Note	**Special Case:** If ***their quadratic*** factor $x^2+"B"x+"C"$ **can** be factorised then allow dM1 for correct factorisation leading to $x=...$ Otherwise, give dM0 for applying a method of factorisation to solve their 3TQ = 0.

	Question 2 Notes Continued	
2. (b)	**Note**	**Reminder: Method mark for solving a 3TQ = 0** **Formula:** $Ax^2+Bx+C=0 \Rightarrow$ Attempt to use the correct formula (with values for A, B, C) **Completing the Square:** $x^2+Bx+C=0 \Rightarrow \left(x \pm \frac{B}{2}\right)^2 \pm q \pm C = 0,\ q \neq 0,$ leading to $x = ...$
	Note:	**Comparing coefficients:** $f(x) = (3x+1)(x^2+\alpha x+\beta) \equiv 3x^3-17x^2+33x+13$ $x^2 : 3\alpha + 1 = -17 \Rightarrow \alpha = -6;\ \ z : 3\beta + \alpha = 33 \Rightarrow 3\beta - 6 = 33 \Rightarrow \beta = 13;$ constant : $\beta = 13$ yielding quadratic factor = $x^2-6x+13$
	Note	The solutions $3 \pm 2i$ need to follow on from a correct $x^2-6x+13=0$ or $3x^2-18x+39=0$ in order to gain the final A mark.
	Note	Give final A0 for writing $\frac{6 \pm 4i}{2}$ followed by either $3 \pm 4i$ or $6 \pm 2i$
2. (a) **ALT 1**	**Note**	**Long division:** $x+\frac{1}{3}\ \big|\ 3x^3+kx^2+33x+13$ $3x^3+x^2$ $(k-1)x^2+33x$ $-18x^2-6x$ $39x+13$ $39x+13$ 0 or $3x+1\ \big|\ 3x^3+kx^2+33x+13$ $x^2-6x+13$ $3x^3+x^2$ $(k-1)x^2+33x$ $-18x^2-6x$ $39x+13$ $39x+13$ 0
		$(k-1)--18=0 \Rightarrow k=...$ | Full complete method of dividing by either $x+\frac{1}{3}$ or $(3x+1)$, applying remainder = 0 and solving a relevant equation to find $k = ...$ | M1
		$k=-17$ | $k=-17$ | A1
		(2)
	Note	Give M0 for dividing by either $x-\frac{1}{3}$ or $3x-1$

	Question 2 Notes Continued			
2. (a) **ALT 2**	**Note**	**Long division:** $$\begin{array}{r} x^2 + \left(\frac{k-1}{3}\right)x + \left(\frac{100-k}{9}\right) \\ 3x+1 \,\big|\, 3x^3 + kx^2 + 33x + 13 \\ 3x^3 + x^2 \\ \hline (k-1)x^2 + 33x \\ (k-1)x^2 + \left(\frac{k-1}{3}\right)x \\ \hline \left(\frac{100-k}{3}\right)x + 13 \\ \left(\frac{100-k}{3}\right)x + \left(\frac{100-k}{9}\right) \\ \hline 13 - \left(\frac{100-k}{9}\right) \end{array}$$ or $$\begin{array}{r} 3x^2 + (k-1)x + \left(\frac{100-k}{3}\right) \\ x+\frac{1}{3} \,\big|\, 3x^3 + kx^2 + 33x + 13 \\ 3x^3 + x^2 \\ \hline (k-1)x^2 + 33x \\ (k-1)x^2 + \left(\frac{k-1}{3}\right)x \\ \hline \left(\frac{100-k}{3}\right)x + 13 \\ \left(\frac{100-k}{3}\right)x + \left(\frac{100-k}{9}\right) \\ \hline 13 - \left(\frac{100-k}{9}\right) \end{array}$$		
		$13 - \left(\frac{100-k}{9}\right) = 0 \Rightarrow k = ...$ or $33 - \left(\frac{k-1}{3}\right) = 39 \Rightarrow k = ...$	Full complete method of dividing by either $x+\frac{1}{3}$ or $(3x+1)$, applying remainder = 0 and solving a relevant equation to find $k = ...$	M1
		$\left\{\frac{117-100+k}{9} = 0 \Rightarrow\right\} k = -17$	$k = -17$	A1
				(2)
	Note	Give M0 for dividing by either $x-\frac{1}{3}$ or $3x-1$		