Question Number	Scheme	Notes	Marks
4.	$z_1 = p + 5\text{i},\ z_2 = 9 + 8\text{i},\ z_3 = \dfrac{z_1}{z_2};\ \arg(z_1) = \dfrac{\pi}{3}$		
(a) **Way 1**	$z_3 = \dfrac{(p+5\text{i})}{(9+8\text{i})} \times \dfrac{(9-8\text{i})}{(9-8\text{i})}$	Multiplies numerator **and** denominator of z_3 by $9-8\text{i}$	M1
	$= \dfrac{9p - 8p\text{i} + 45\text{i} + 40}{81+64}$	Applies $\text{i}^2 = -1$ to give either • a correct expression in terms of p for the numerator **or** • a correct numerical expression or value for the denominator	A1
	$= \dfrac{9p+40}{145} + \left(\dfrac{-8p+45}{145}\right)\text{i}$	Correct answer written in the form $x + \text{i}y$ o.e. **or** writes a correct $x = \dfrac{9p+40}{145}$, $y = \dfrac{-8p+45}{145}$	A1
			(3)
(a) **Way 2**	$z_3 = \dfrac{(p+5\text{i})}{(9+8\text{i})} \times \dfrac{(-9+8\text{i})}{(-9+8\text{i})}$	Multiplies numerator **and** denominator of z_3 by $-9+8\text{i}$	M1
	$= \dfrac{-9p + 8p\text{i} - 45\text{i} - 40}{-81-64}$	Applies $\text{i}^2 = -1$ to give either • a correct expression in terms of p for the numerator **or** • a correct numerical expression or value for the denominator	A1
	$= \dfrac{-9p-40}{-145} + \left(\dfrac{8p-45}{-145}\right)\text{i}$	Correct answer written in the form $x + \text{i}y$ o.e. **or** writes a correct $x = \dfrac{-9p-40}{-145}$ and $y = \dfrac{8p-45}{-145}$	A1
			(3)
(b)	$\left\{\lvert z_2\rvert = \sqrt{9^2+8^2} \Rightarrow\right\}\ \lvert z_2\rvert = \sqrt{145}$	$\sqrt{145}$	B1
			(1)
(c)(i) **Way 1**	$\left\{\arg(z_1) = \dfrac{\pi}{3} \Rightarrow\right\}$		
	e.g. $\arctan\left(\dfrac{5}{p}\right) = \dfrac{\pi}{3}$ or $\tan\left(\dfrac{\pi}{3}\right) = \dfrac{5}{p}$ or $\sqrt{3} = \dfrac{5}{p}$	Uses trigonometry to form a correct equation in p	M1
	$p = \dfrac{5}{\sqrt{3}}$ or $\dfrac{5}{3}\sqrt{3}$ or $\sqrt{\dfrac{25}{3}}$	Correct exact value for p **Note:** You can apply isw	A1
(c)(i) **Way 2**	$\left\{z_1 = \sqrt{p^2+25}\left(\cos\dfrac{\pi}{3} + \text{i}\sin\dfrac{\pi}{3}\right) = p + 5\text{i} \Rightarrow\right\}$		
	e.g. $\sqrt{p^2+25}\left(\cos\dfrac{\pi}{3}\right) = p$ or $\sqrt{p^2+25}\left(\sin\dfrac{\pi}{3}\right) = 5$	Uses trigonometry to form a correct equation in p	M1
	$p = \dfrac{5}{\sqrt{3}}$ or $\dfrac{5}{3}\sqrt{3}$ or $\sqrt{\dfrac{25}{3}}$	Correct exact value for p **Note:** You can apply isw	A1
(ii)	• $\lvert z_3\rvert = \dfrac{\lvert z_1\rvert}{\lvert z_2\rvert} = \dfrac{\sqrt{\left(\dfrac{5}{\sqrt{3}}\right)^2 + (5)^2}}{\sqrt{145}} = \dfrac{\sqrt{\dfrac{100}{3}}}{\sqrt{145}}$ • $z_3 = \dfrac{8+3\sqrt{3}}{29} + \dfrac{27-8\sqrt{3}}{87} \Rightarrow \lvert z_3\rvert = \sqrt{\left(\dfrac{8+3\sqrt{3}}{29}\right)^2 + \left(\dfrac{27-8\sqrt{3}}{87}\right)^2}$		
	$\lvert z_3\rvert = \dfrac{10}{\sqrt{435}}$ or $\dfrac{10}{435}\sqrt{435}$ or $\dfrac{2}{87}\sqrt{435}$ or $\dfrac{2\sqrt{435}}{87}$	Correct exact answer written in the form $\dfrac{a}{\sqrt{b}}$ or $c\sqrt{b}$; $a,b \in \mathbb{Z}, c \in \mathbb{Q}$	B1
	Note: Give B1 for $\lvert z_3\rvert = \sqrt{\dfrac{20}{87}}$		**(3)**
			7

	Question 4 Notes	
4. (a)	**Note**	Give 2nd A0 for $z_3 = \frac{9p+40}{81+64} + \left(\frac{-8p+45}{81+64}\right)\text{i}$ without reference to $z_3 = \frac{9p+40}{145} + \left(\frac{-8p+45}{145}\right)\text{i}$
	Note	$\frac{9p+40+(45-8p)\text{i}}{145}$ is not considered to be in the form $x+\text{i}y$
	Note	Allow final A1 for $z_3 = \frac{9p}{145} + \frac{8}{29} + \left(\frac{9}{29} - \frac{8p}{145}\right)\text{i}$
	Note	Allow final A1 for $z_3 = \frac{9p+40}{145} - \left(\frac{8p-45}{145}\right)\text{i}$
	Note	y written as $y = \left(\frac{-8p+45}{145}\right)\text{i}$ is incorrect
	Note	M1 A1 can be implied for writing $z_3 = \frac{(p+5\text{i})}{(9+8\text{i})} = \frac{9p-8p\text{i}}{145} + \frac{8+9\text{i}}{29}$ and final A1 is then given for $z_3 = \frac{9p}{145} + \frac{8}{29} + \left(\frac{9}{29} - \frac{8p}{145}\right)\text{i}$
(b)	**Note**	You can apply isw after seeing $\sqrt{145}$
	Note	Give B0 for writing $12, 12.0$ or awrt 12.0 without reference to $\sqrt{145}$
(c)(i)	**Note**	Give M1 for any of $\arctan\left(\frac{5}{p}\right) = 60$, $\tan 60 = \frac{5}{p}$, $\arctan\left(\frac{p}{5}\right) = \frac{\pi}{6}$, $\tan 30 = \frac{p}{5}$
	Note	Give M1 A0 for $p = 2.88$ (truncated) or $p =$ awrt 2.89 without reference to a correct exact value
	Note	Give A0 for $p = \pm\frac{5}{\sqrt{3}}$ with no evidence of rejecting the negative value of p
(c)(ii)	**Note**	Allow B1 for $\lvert z_3 \rvert = \frac{\sqrt{1740}}{87}$