Question Number	Scheme	Notes	Marks
5.	$f(x) = x^4 - 12x^{\frac{3}{2}} + 7 \; ; \quad x \geqslant 0$		
(a) **Way 1**	$f(2) = -10.9411255...$ $f(3) = \; 25.64617093...$	Attempts to evaluate both $f(2)$ and $f(3)$ **and** either $f(2) = -10$ (truncated) or awrt -11 **or** $f(3) = 25$ (truncated) or awrt 26	M1
	Sign change {negative, positive} {and $f(x)$ is continuous} therefore a root $\{\alpha\}$ exists in the interval $\{[2, 3]\}$	Both values correct awrt (or truncated) to 2 sf, reason and a valid conclusion	A1 **cso**
			(2)
(b)	$f'(x) = 4x^3 - 18x^{\frac{1}{2}}$	At least one of either $x^4 \to \pm Ax^3$ or $-12x^{\frac{3}{2}} \to \pm Bx^{\frac{1}{2}}$; $A, B \neq 0$	M1
		Correct differentiation, which can be un-simplified or simplified	A1
	$\left\{\alpha \simeq 2.5 - \dfrac{f(2.5)}{f'(2.5)} \Rightarrow\right\} \alpha \simeq 2.5 - \dfrac{(2.5)^4 - 12(2.5)^{\frac{3}{2}} + 7}{4(2.5)^3 - 18(2.5)^{\frac{1}{2}}}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using the applied $f(2.5)$ and their applied $f'(2.5)$	dM1
	$\left\{\alpha \simeq 2.5 - \dfrac{-1.3716649...}{34.0395011...} = 2.5 + 0.0402962...\right\}$		
	$\alpha = 2.54$ (2 dp)	**dependent on all 3 previous marks** 2.54 on first iteration (Ignore any subsequent iterations)	A1 **cao** **cso**
	Correct differentiation followed by 2.54 (with no working seen) scores full marks in part (b)		**(4)**
(c) **Way 1**	$f(2.535) = -0.137392933...$ $f(2.545) = \; 0.231219419...$	Chooses a suitable interval $[x_L, x_U]$ for x, which is within ± 0.005 and either side of their answer to (b) and attempts to find either $f(x_L)$ or $f(x_U)$	M1
	Sign change {negative, positive} {and $f(x)$ is continuous} therefore (a root) $\alpha = 2.54$ {2 dp}	Both values correct awrt 1 sf, reason and a valid conclusion	A1
			(2)
(c) Way 2	**Condoned Method: Applying Newton-Raphson again.** E.g. Using $\alpha = 2.54, 2.5402962...$		
	• $\alpha \simeq 2.54 - \dfrac{0.046101609...}{36.8609766...} = 2.538751631...$ • $\alpha \simeq 2.5402962... - \dfrac{0.05693746...}{36.88822382...} = 2.538752436...$ So $\alpha = 2.54$ (2 dp)	Evidence of applying Newton-Raphson for a second time on their answer to part (b)	M1
		Obtains either a truncated 2.538 or awrt 2.539 and a valid conclusion	A1
	Note: Work for Way 2 can be recovered in part (b)		**(2)**
			8

	Question 5 Notes	
5. (a)	**Note**	**Way 1: A1, correct solution only** Required to state **both** values for $f(2)$ and $f(3)$ correct awrt (or truncated) to 2 sf along with **a reason and a conclusion.** Reference to change of sign **or** e.g. $f(2) \times f(3) < 0$ **or** $f(2) < 0 < f(3)$ **or** a diagram **or** < 0 and > 0 **or** one negative, one positive are sufficient reasons. There must be a conclusion, e.g. $\{x \text{ or}\} \; \alpha \in [2, 3]$ or $\{x \text{ or}\} \; \alpha \in (2, 3)$ or root lies between 2 and 3. Ignore the presence or absence of any reference to continuity.
	Note	A minimal acceptable reason and conclusion is "change of sign, so $\alpha \in [2, 3]$" **or** "change of sign, so root is between 2 and 3" **or** "change of sign, so root" **or** "$f(2) = -10.9 < 0, \; f(3) = 25.6 > 0,$ so root" **or** "change of sign, so in the interval"

	Question 5 Notes Continued	
5. (a)	**Note**	Give final A0 for writing as their conclusion "root lies between $f(2)$ and $f(3)$"
5. (a)	**Note**	**ALT** The root of $f(x)=0$ is $2.5388...$, so they can choose x_1 which is less than $2.5388...$, and choose x_2 which is greater than $2.5388...$ with both x_1 and x_2 lying in the interval $[2, 3]$. **M1:** Finds $f(x_1)$ and $f(x_2)$ with one of these values correct awrt (or truncated) to 2 sf **A1:** Both values correct awrt (or truncated) to 2 sf, reason (e.g. sign change) and conclusion
	Note	**Helpful Table** x | $f(x)$ 2 | −10.9411255... 2.1 | −10.0701694... 2.2 | −8.731928012... 2.3 | −6.873372451... 2.4 | −4.439168148... 2.5 | −1.371664903... 2.6 | 2.389111651... 2.7 | 6.90546741... 2.8 | 12.24204622... 2.9 | 18.46583545... 3 | 25.64617093...
(b)	**dM1**	This mark can be implied by applying at least one correct *value* of either $f(2.5)$ or their $f'(2.5)$ (where $f'(2.5)$ is found using their $f'(x)$) to awrt 2 significant figures in $2.5-\frac{f(2.5)}{f'(2.5)}$. So ***just writing*** $2.5-\frac{f(2.5)}{f'(2.5)}$ with an incorrect ft answer on their $f'(2.5)$ scores dM0 A0.
	Note	Allow M1 A1 dM1 A1 for $2.5-\frac{f(2.5)}{f'(2.5)}=2.54$ with no algebraic differentiation
	Note	Allow M1 A1 dM1 A1 for correct answer 2.54 given with no other working
	Note	You can imply the M1 A1 marks for the absence of algebraic differentiation by either • $f'(2.5)=4(2.5)^3-18(2.5)^{\frac{1}{2}}$ • $f'(2.5)$ applied correctly in $\alpha \simeq 2.5-\frac{(2.5)^4-12(2.5)^{\frac{3}{2}}+7}{4(2.5)^3-18(2.5)^{\frac{1}{2}}}$ • $f'(2.5)$ = awrt 34
	Note	**Differentiating INCORRECTLY to give** $f'(x)=4x^3+18x^{\frac{1}{2}}$ leads to $\alpha \simeq 2.5-\frac{-1.3716649...}{90.9604989...}=2.51507978...=2.52$ (2 dp) **This response should be given M1 A0 dM1 A0**
	Note	**Differentiating INCORRECTLY to give** $f'(x)=4x^3+18x^{\frac{1}{2}}$ and $\alpha \simeq 2.5-\frac{f(2.5)}{f'(2.5)}=2.52$ is M1 A0 dM1 A0

<table>
<tr><th></th><th colspan="2">Question 5 Notes Continued</th></tr>
<tr><td rowspan="5">5. (c)</td><td>Note</td><td>If they obtain a correct answer 2.54 by an incorrect method in part (b) then M1 A1 is allowed in part (c).</td></tr>
<tr><td>Note</td><td>Way 1: A1, correct solution only
Required to state both values for $f(x_L)$ and $f(x_U)$ correct awrt (or truncated) to 1 sf along with a reason and a conclusion. Reference to change of sign or e.g. $f(2.535)\times f(2.545)<0$ or $f(2.535)<0<f(2.545)$ or a diagram or <0 and >0 or one negative, one positive are sufficient reasons. There must be a (minimal, not incorrect) conclusion e.g. $\alpha=2.54$, root (or α to part (b)) is correct, QED or □ are all acceptable. Ignore the presence or absence of any reference to continuity.</td></tr>
<tr><td>Note</td><td>A minimal acceptable reason and conclusion is any of
“change of sign, hence root”
“change of sign, so $\alpha=2.54$”
“change of sign, so $x=2.54$”
“change of sign, so α is correct {to 2 decimal places}”
“$f(2.535)=-0.1<0,\ f(2.545)=0.2>0,$ so root”
“$f(2.535)=-0.1<0,\ f(2.545)=0.2>0,$ so $\alpha=2.54$”</td></tr>
<tr><td>Note</td><td>No explicit reference to 2 decimal places is necessary for the conclusion</td></tr>
<tr><td>Note</td><td>Give A0 for stating “root is in between 2.535 and 2.545” or “root lies in the given interval” without reference to either $\alpha=2.54$, root (or α to part (b)) is correct, QED or □</td></tr>
<tr><td rowspan="2">(c)</td><td>Note</td><td><u>Way 1: ALT</u>
The root of $f(x)=0$ is 2.5388..., so they can choose x_L which is less than 2.5388..., and choose x_U which is greater than 2.5388... with both x_L and x_U lying in the interval $[2.535,\ 2.545]$ and evaluate $f(x_L)$ and $f(x_U)$
M1: Chooses a suitable interval $[x_L,\ x_U]$ and attempts to find either $f(x_L)$ or $f(x_U)$
A1: Both values correct awrt (or truncated) to 1 sf, reason (e.g. sign change) and conclusion</td></tr>
<tr><td>Note</td><td><u>Helpful Table</u>
<table><tr><th>x</th><th>$f(x)$</th></tr><tr><td>2.535</td><td>−0.137392933…</td></tr><tr><td>2.536</td><td>−0.100854301…</td></tr><tr><td>2.537</td><td>−0.064244144…</td></tr><tr><td>2.538</td><td>−0.027562401…</td></tr><tr><td>2.539</td><td>0.00919099…</td></tr><tr><td>2.54</td><td>0.046016091…</td></tr><tr><td>2.541</td><td>0.082912964…</td></tr><tr><td>2.542</td><td>0.119881671…</td></tr><tr><td>2.543</td><td>0.156922274…</td></tr><tr><td>2.544</td><td>0.194034836…</td></tr><tr><td>2.545</td><td>0.231219419…</td></tr></table></td></tr>
<tr><td rowspan="2">(c)
Way 2</td><td>Note</td><td>If $\alpha=2.54$ in part (b), then give M1 A1 in part (c) for any of
“$\alpha_2=2.538\Rightarrow\alpha_2=2.54$”
“$\alpha_2=2.539\Rightarrow\alpha_2=2.54$”
“$\alpha_2=2.539$, so answer to part (b) is correct”</td></tr>
<tr><td>Note</td><td>If $\alpha=2.54$ in part (b), then give M1 A0 in part (c) for writing “$\alpha\simeq 2.54-\dfrac{f(2.54)}{f'(2.54)}=2.54$”</td></tr>
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