Question Number	Scheme	Notes	Marks
9.	$f(n)=7^n(3n+1)-1$ is a multiple of 9	$u_1=2,\ u_2=6,\ u_{n+2}=3u_{n+1}-2u_n \Rightarrow u_n=2(2^n-1)$	
(i) **Way 1**	$f(1)=7(4)-1=27$ {is a multiple of 9}	$f(1)=27$ is the minimum	B1
	$f(k+1)-f(k)=\underline{7^{k+1}(3(k+1)+1)-1}-(7^k(3k+1)-1)$	Attempts $f(k+1)-f(k)$	M1
		A correct expression for $\underline{f(k+1)}$	A1
	$=7^{k+1}(3k+4)-1-7^k(3k+1)+1=7^k(21k+28)-7^k(3k+1)$		
	$=18k(7^k)+27(7^k)$ or $7^k(18k+27)$	**dependent on the previous M mark** Uses correct algebra to achieve an expression where each term is an obvious multiple of 9	dM1
	$f(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $f(k+1)=18k(7^k)+27(7^k)+f(k)$	Correct algebra leading to either e.g. $f(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $f(k+1)=18k(7^k)+27(7^k)+f(k)$	A1
	If the result is $\underline{\text{true for } n=k}$, then it is $\underline{\text{true for } n=k+1}$. As the result has been shown to be $\underline{\text{true for } n=1}$, then the result $\underline{\text{is true for all } n}$ $(\in\mathbb{Z}^+)$		A1 cso
			(6)
(i) **Way 2**	$f(1)=7(4)-1=27$ {is a multiple of 9}	$f(1)=27$ is the minimum	B1
	$f(k+1)=7^{k+1}(3(k+1)+1)-1$	Attempts $f(k+1)$	M1
		A correct expression for $f(k+1)$	A1
	$=7^{k+1}(3k+4)-1=7^k(21k+28)-1$		
	$=18k(7^k)+27(7^k)+7^k(3k+1)-1$ **or** $=(7^k)(18k+27)+7^k(3k+1)-1$ **or** $=9(7^k)(2k+3)+7^k(3k+1)-1$	**dependent on the previous M mark** Uses correct algebra to express $f(k+1)=g(k)+7^k(3k+1)-1$ or $f(k+1)=g(k)+f(k)$ where each term in $g(k)$ is an obvious multiple of 9	dM1
		Correct algebra leading to either e.g. $f(k+1)=9(7^k)(2k+3)+7^k(3k+1)-1$ or $f(k+1)=18k(7^k)+27(7^k)+f(k)$	A1
	If the result is $\underline{\text{true for } n=k}$, then it is $\underline{\text{true for } n=k+1}$. As the result has been shown to be $\underline{\text{true for } n=1}$, then the result $\underline{\text{is true for all } n}$ $(\in\mathbb{Z}^+)$		A1 cso
			(6)
(ii)	$\{n=1,\}\ u_1=2(2^1-1)=2$; $\{n=2,\}\ u_2=2(2^2-1)=6$	Checks that the general formula works for either u_1 or u_2	M1
		Checks that the general formula works for both u_1 and u_2	A1
	$\{u_{k+2}=3u_{k+1}-2u_k \Rightarrow\}$ $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))$	Finds u_{k+2} by attempting to substitute $u_{k+1}=2(2^{k+1}-1)$ and $u_k=2(2^k-1)$ into $u_{k+2}=3u_{k+1}-2u_k$ Condone one slip	M1
	$\{u_{k+2}\}=6(2^{k+1})-6-4(2^k)+4$		
	$\{u_{k+2}\}=3(2^{k+2})-2^{k+2}-2$	Valid evidence of working in the same power of 2	M1
	$=2(2^{k+2})-2=2(2^{k+2}-1)$	***Uses algebra*** in a complete method to achieve this result with no errors	A1
	If the result is $\underline{\text{true for } n=k \text{ and for } n=k+1}$, then it is $\underline{\text{true for } n=k+2}$. As the result has been shown to be $\underline{\text{true for } n=1 \text{ and } n=2}$, then the result $\underline{\text{is true for all } n}$ $(\in\mathbb{Z}^+)$		A1 **cso**
			(6)
			12

		Question 9 Notes
9. (i)	**Note**	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (i)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	Shows $f(k+1)-f(k)=7^k(18k+27)$ **or** $f(k+1)-f(k)=9(7^k)(2k+3)$ and writing if $f(k+1)-f(k)=9(7^k)(2k+3)$ o.e. is a multiple of 9 then $f(k+1)$ is a multiple of 9 is acceptable for the penultimate A mark in part (i). This means that the final A mark can potentially be available.
	Note	Only showing $f(k+1)=7f(k)+6+21(7^k)$ (see Way 4) does not get the final dM mark because $6+21(7^k)$ is not an obvious multiple of 9
	Note	Allow dM1 for obtaining e.g. $f(k+1)-f(k)=18k(7^k)-27(7^k)$ or $f(k+1)-f(k)=7^k(18k-27)$
	Note	Allow dM1 for obtaining $f(k+1)=18k(7^k)-27(7^k)+7^k(3k+1)-1$ or $f(k+1)=9(7^k)(2k-3)+f(k)$
(ii)	**Note**	**1st M1:** At least one check is correct. **1st A1:** Both checks are correct • Check 1: Shows $u_1=2$ by writing an intermediate step of e.g. $2(2^1-1)$ or 2×1 • Check 2: Shows $u_2=6$ by writing an intermediate step of e.g. $2(2^2-1)$ or 2×3
	Note	Ignore $u_3=3u_2-2u_1=3(6)-2(2)=14$ as part of their solution to (ii)
	Note	Ignore $\{n=3,\}$ $u_2=2(2^3-1)=14$ as part of their solution to (ii)
	Note	Valid evidence of working in the same power of 2 includes: • $6(2^{k+1})-4(2^k)\to6(2^{k+1})-2(2^{k+1})$ or $2(3(2^{k+1})-2^{k+1})$ • $3(2(2^{k+1}))-2(2(2^k))\to3(2^{k+2})-(2^{k+2})$ • $3(2(2^{k+1}))-2(2(2^k))\to12(2^k)-4(2^k)$ • $6(2^{k+1})-4(2^k)\to8(2^k)$ (by implication) • $6(2^{k+1})-4(2^k)\to4(2^{k+1})$ (by implication)
	Note	Writing $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))=2(2^{k+2}-1)$ is 2nd M1, 3rd M0, 2nd A0
	Note	Showing $\{\text{RHS}=\}$ $u_{k+2}=2(2^{k+2}-1)=8(2^k)-2$ and writing $\{\text{LHS}=\}$ $u_{k+2}=3(2(2^{k+1}-1))-2(2(2^k-1))$ and using valid algebra to show that $u_{k+2}=8(2^k)-2$ $\{=\text{RHS}\}$ is fine for the 2nd M, 3rd M and 2nd A marks
	Note	**Final A1** is dependent on all previous marks being scored. It is gained by candidates conveying the ideas of **all** four underlined points **in part (ii)** **either** at the end of their solution **or** as a narrative in their solution.
	Note	"Assume for $n=k$, $u_k=2(2^k-1)$ and for $n=k+1$, $u_{k+1}=2(2^{k+1}-1)$" satisfies the requirement "true for $\boldsymbol{n=k}$ and $n=k+1$"
	Note	"For $n\in\mathbb{Z}^+$, $u_n=2(2^n-1)$ " satisfies the requirement "true for all n"
	Note	Full marks in (ii) can be obtained for an equivalent proof where e.g. • $n=k, n=k+1, \to n=k-2, n=k-1$; i.e. $k\equiv k-2$ • $n=k, n=k+1, \to n=k-1, n=k$; i.e. $k\equiv k-1$
(i), (ii)	**Note**	Allow as part of their conclusion "true for all positive values of n"
	Note	Allow as part of their conclusion "true for all values of n"
	Note	Allow as part of their conclusion "true for all $n\in\mathbb{N}$"
	Note	Condone referring to n as any integer in their conclusion for the final A1
	Note	Condone $n\in\mathbb{Z}^*$ as part of their conclusion for the final A1
	Note	Referring to n as a real number their conclusion is final A0

Question Number	Scheme	Notes	Marks
9.	$f(n)=7^n(3n+1)-1$ is a multiple of 9; $P\in\mathbb{Z}^+$		
(i) **Way 3**	$f(1)=7(4)-1=27$ {is a multiple of 9}	$f(1)=27$ is the minimum	B1
	$f(k+1)-(9P+1)f(k)$	Attempts $f(k+1)-(9P+1)f(k)$	M1
	$=\underline{7^{k+1}(3(k+1)+1)-1}-(9P+1)(7^k(3k+1)-1)$	A correct expression for $\underline{f(k+1)}$	A1
	$=7^k(21k+28-(9P+1)(3k+1))-1+9P+1$		
	$=7^k(21k+28-(27Pk+9P+3k+1))-1+9P+1$		
	$=7^k(21k+28-27Pk-9P-3k-1)+9P$		
	$=7^k(18k-27Pk-9P+27)+9P$	**dependent on the previous M mark** Uses correct algebra to achieve an expression where each term is an obvious multiple of 9	dM1
	$f(k+1)=7^k(18k-27PK-9P+27)+9P+(9P+1)f(k)$	Achieves a correct result for $f(k+1)=...$	A1
	If the result is $\underline{\text{true for } n=k}$, then it is $\underline{\text{true for } n=k+1}$. As the result has been shown to be $\underline{\text{true for } n=1}$, then the result $\underline{\text{is true for all } n}$ $(\in\mathbb{Z}^+)$		A1 cso
			(6)
	Note: $P=0\Rightarrow f(k+1)-f(k)=7^k(18k+27)$ $P=1\Rightarrow f(k+1)-10f(k)=7^k(18-9k)+9$ $P=2\Rightarrow f(k+1)-19f(k)=7^k(9-36k)+18$ $P=3\Rightarrow f(k+1)-28f(k)=7^k(-63k)+27=27-9k(7^{k+1})$		

Question Number	Scheme	Notes	Marks
9.	$f(n) = 7^n(3n+1)-1$ is a multiple of 9		
(i) **Way 4**	$f(1) = 7(4)-1 = 27$ {is a multiple of 9}	$f(1) = 27$ is the minimum	B1
	$f(k+1) = 7^{k+1}(3(k+1)+1)-1$	Attempts $f(k+1)$	M1
		A correct expression for $f(k+1)$	A1
	$= 7(7^k)(3k+3+1)-1$		
	$= 7(7^k)(3k+1)+3(7)(7^k)-1$		
	$= 7[(7^k)(3k+1)-1] + 7 + 21(7^k) - 1$ $= 7f(k) + 6 + 21(7^k)$ Let $g(n) = 6 + 21(7^n)$ $g(1) = 6 + 21(7^1) = 153$ {is a multiple of 9} {Assume the result is true for $n = k$} $g(k+1) = 6 + 21(7^{k+1})$ $= 6 + 147(7^k)$ $= 6 + 21(7^k) + 126(7^k)$ or $= g(k) + 9(14)(7^k)$	**dependent on the previous M mark** Uses correct algebra to express $f(k+1) = \alpha(7^k(3k+1)-1) + g(k)$ or $f(k+1) = \alpha f(k) + g(k);\ \alpha \neq 0$ and uses correct algebra to achieve an expression for $g(k+1)$ where each term is an obvious multiple of 9	M1
		Correct algebra leading to $f(k+1) = 7f(k) + 6 + 21(7^k)$ o.e. and $g(k+1) = 6 + 21(7^k) + 126(7^k)$ where $g(n) = 6 + 21(7^n)$	A1
	Proves that $g(n) = 6 + 21(7^n)$ is a multiple of 9 and proves that for $f(n)$ if the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n $(\in \mathbb{Z}^+)$		A1 cso
			(6)
	Note: An alternative Way 4 method shows • $f(k+1) = 7f(k) + 6 + 21(7^k) = 7f(k) + 9(7^k+1) + 3(7^k) - 3$ • Defines $g(n) = 3(7^n) - 3$ and proceeds to show that $g(n)$ is also a multiple of 9		

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