Question Number	Scheme	Notes	Marks
8(i)	$\sum_{r=1}^{n}\frac{2r^2-1}{r^2(r+1)^2}=\frac{n^2}{(n+1)^2}$		
	$\frac{2(1)^2-1}{1^2(1+1)^2}=\frac{1}{4},\ \frac{1^2}{(1+1)^2}=\frac{1}{4}$	Getting $\frac{1}{2^2}$ or $\frac{1}{4}$ from each is the minimum	B1
	Assume $\sum_{r=1}^{k}\frac{2r^2-1}{r^2(r+1)^2}=\frac{k^2}{(k+1)^2}$		
	$\sum_{r=1}^{k+1}\frac{2r^2-1}{r^2(r+1)^2}=\frac{k^2}{(k+1)^2}+\frac{2(k+1)^2-1}{(k+1)^2(k+2)^2}$ Assumes the result is true for say $n = k$ and adds the next term		M1
	$\frac{k^2(k+2)^2+2(k+1)^2-1}{(k+1)^2(k+2)^2}$	Attempts common denominator	dM1
		Correct expression	A1
	$\frac{k^4+4k^3+4k^2+2k^2+4k+1}{(k+1)^2(k+2)^2}=\frac{k^4+4k^3+6k^2+4k+1}{(k+1)^2(k+2)^2}=\frac{(k+1)^4}{(k+1)^2(k+2)^2}$		
	$\frac{(k+1)^2}{(k+2)^2}$	Achieves this result with intermediate working and no errors	A1
	If the result is <u>true for</u> $n=k$, then it is <u>true for</u> $n=k+1$. As the result has been shown to be <u>true for</u> $n=1$, then the result is <u>true for all</u> n.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
			(6)
(ii)	$f(n)=12^n+2\times5^{n-1}$		
	$f(1)=12+2\times1=14$	This is sufficient	B1
	$f(k+1)=12^{k+1}+2\times5^k$	Attempt f(k + 1)	M1
	$f(k+1)-f(k)=12^{k+1}+2\times5^k-12^k-2\times5^{k-1}$ $f(k+1)-f(k)=11\times12^k+22\times5^{k-1}+10\times5^{k-1}-24\times5^{k-1}$	Working with f(k + 1) – f(k)	
	$=11\times\left(12^k+2\times5^{k-1}\right)-14\times5^{k-1}$	$11\times\left(12^k+2\times5^{k-1}\right)$ or 11f(k)	A1
		$-14\times5^{k-1}$	A1
	$f(k+1)=12f(k)-14\times5^{k-1}$	Makes f(k + 1) the subject **Dependent on at least one of the A marks**	dM1
	If the result is <u>true for</u> $n=k$, then it is <u>true for</u> $n=k+1$. As the result has been shown to be <u>true for</u> $n=1$, then the result is <u>true for all</u> n.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso
			(6)
			Total 12

<table>
<tr><td rowspan="7">ALT 1</td><td>$f(1)=12+2\times1=14$</td><td colspan="2">This is sufficient</td><td>B1</td></tr>
<tr><td>$f(k+1)=12^{k+1}+2\times5^{k}$</td><td colspan="2">Attempt f(k+1)</td><td>M1</td></tr>
<tr><td colspan="3">$f(k+1)=12\left(12^{k}+2\times5^{k-1}\right)+2\times5\times5^{k-1}-12\times2\times5^{k-1}$</td><td></td></tr>
<tr><td rowspan="2">$f(k+1)=12\left(12^{k}+2\times5^{k-1}\right)-14\times5^{k-1}$</td><td>$12\left(12^{k}+2\times5^{k-1}\right)$ or $12f(k)$</td><td>A1</td></tr>
<tr><td>$-14\times5^{k-1}$</td><td>A1</td></tr>
<tr><td colspan="2">$f(k+1)=12f\left(k\right)-14\times5^{k-1}$</td><td>Dependent on at least one of the A marks</td><td>dM1</td></tr>
<tr><td colspan="2">If the result is <u>true for $n=k$,</u> then it is <u>true for $n=k+1$.</u> As the result has been shown to be <u>true for $n=1$,</u> then the result is <u>true for all n</u>.</td><td colspan="2">The underlined features should be seen. Some may appear earlier in the solution.</td><td>A1cso</td></tr>
<tr><td rowspan="7">ALT 2</td><td>$f(1)=12+2\times1=14$</td><td colspan="2">This is sufficient</td><td>B1</td></tr>
<tr><td>Let $12^{k}+2\times5^{k-1}=7M$</td><td colspan="2"></td><td></td></tr>
<tr><td>$f(k+1)=12^{k+1}+2\times5^{k}$</td><td colspan="2">Attempt f(k+1)</td><td>M1</td></tr>
<tr><td>$f(k+1)=12\left(7M-2\times5^{k-1}\right)+2\times5^{k}$</td><td colspan="2">OR: $f(k+1)=5(7M)+7\times12^{k}$</td><td></td></tr>
<tr><td rowspan="2">$f(k+1)=84M-14\times5^{k-1}$
OR: $f(k+1)=35M+7\times12^{k}$</td><td>$84M$</td><td>OR: $35M$</td><td>A1</td></tr>
<tr><td>$-14\times5^{k-1}$</td><td>$+7\times12^{k}$</td><td>A1</td></tr>
<tr><td>$f(k+1)=12f\left(k\right)-14\times5^{k-1}$
OR:
$f(k+1)=5f(k)+7\times12^{k}$</td><td colspan="2">Dependent on at least one of the A marks</td><td>dM1</td></tr>
<tr><td>If the result is <u>true for $n=k$,</u> then it is <u>true for $n=k+1$.</u> As the result has been shown to be <u>true for $n=1$,</u> then the result is <u>true for all n</u>.</td><td colspan="2">The underlined features should be seen. Some may appear earlier in the solution.</td><td>A1cso</td></tr>
<tr><td rowspan="6">ALT 3</td><td colspan="2">$f(1)=12+2\times1=14$</td><td>This is sufficient</td><td>B1</td></tr>
<tr><td colspan="2">$f(k+1)=12^{k+1}+2\times5^{k}$</td><td>Attempts f($k$ + 1)</td><td>M1</td></tr>
<tr><td colspan="3">Working with $f(k+1)-mf(k)$
$f(k+1)-mf(k)=12^{k+1}+2\times5^{k}-m\left(12^{k}+2\times5^{k-1}\right)$,
$f(k+1)-f(k)=\left(12-m\right)\times12^{k}+2\times\left(12-m\right)\times5^{k-1}+10\times5^{k-1}-24\times5^{k-1}$</td><td></td></tr>
<tr><td rowspan="2">$=\left(12-m\right)\times\left(12^{k}+2\times5^{k-1}\right)-14\times5^{k-1}$</td><td>$\left(12-m\right)\times\left(12^{k}+2\times5^{k-1}\right)$
or $\left(12-m\right)f(k)$</td><td>A1</td></tr>
<tr><td>$-14\times5^{k-1}$</td><td>A1</td></tr>
<tr><td colspan="2">$f\left(k+1\right)=12f\left(k\right)-14\times5^{k-1}$</td><td>Makes f($k$ + 1) the subject
Dependent on at least one of the A marks</td><td>dM1</td></tr>
<tr><td colspan="2">If the result is <u>true for $n=k$,</u> then it is <u>true for $n=k+1$.</u> As the result has been shown to be <u>true for $n=1$,</u> then the result is <u>true for all n</u>.</td><td colspan="2">The underlined features should be seen. Some may appear earlier in the solution.</td><td>A1cso</td></tr>
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ALT 4	$f(1) = 12 + 2 \times 1 = 14$	This is sufficient	B1
	$f(k+1) = 12^{k+1} + 2 \times 5^k$	Attempts f(k + 1)	M1
	$f(k+1) - 5f(k) = 12^{k+1} + 2 \times 5^k - 5\left(12^k + 2 \times 5^{k-1}\right)$	Working with f(k + 1) – 5f(k)	
	$= 7 \times 12^k + 2 \times 5^k - 2 \times 5^k$	7×12^k	A1
		$2 \times 5^k - 2 \times 5^k$ (or zero)	A1
	$f(k+1) = 5f(k) + 7 \times 12^k$	Makes f(k + 1) the subject **Dependent on at least one of the A marks**	dM1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.	The underlined features should be seen. Some may appear earlier in the solution.	A1cso

NOTES:

Part (i)
This approach may be seen:
Assume result is true for $n = k$ and $n = k + 1$
Subtract: (sum to (k + 1) terms) minus (sum to k terms)
Show that this is equal to the (k + 1)th term
Please send any such response to Review.

Part (ii)
Apart from the given alternatives, other versions will work and can be marked equivalently.
If in any doubt, send to Review.

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