Question Number	Scheme	Marks
9(i)	$n=1 \quad u_1 = 3\times\frac{2}{3}-1=1$ (so true for $n = 1$ (†))	B1
	Assume true for $n=k$ ie $u_k = 3\left(\frac{2}{3}\right)^k - 1$ (†)	
	$u_{k+1} = \frac{1}{3}(2u_k - 1) = \frac{1}{3}\left(2\left(3\left(\frac{2}{3}\right)^k - 1\right) - 1\right) = \frac{1}{3}\left(6\left(\frac{2}{3}\right)^k - 2 - 1\right)$	M1A1
	$= \frac{1}{3}\left(2\times 3\left(\frac{2}{3}\right)^{k+1} \times \left(\frac{3}{2}\right) - 2 - 1\right)$	dM1
	$= \frac{1}{3}\times 2\times 3\left(\frac{2}{3}\right)^{k+1} \times \left(\frac{3}{2}\right) + \frac{1}{3}(-2-1)$	
	$= 3\left(\frac{2}{3}\right)^{k+1} - 1$	A1
	$\therefore$ if true for $n=k$, also true for $n=k+1$ (†)	
	$\left(\text{True for } n=1\right)$ so $u_n = 3\left(\frac{2}{3}\right)^n - 1$ is true for all $n \in \mathbb{Z}^+$	A1cso (6)
(ii)	$\text{f}(1) = 2^3 + 3^3 = 8 + 27 = 35$ (Multiple of 7) (so true for $n = 1$ (†))	B1
	Assume $\text{f}(k)$ is a multiple of 7 $\text{f}(k) = 2^{k+2} + 3^{2k+1}$ is a multiple of 7 (†)	
	$\text{f}(k+1) - M\text{f}(k) = 2^{k+3} + 3^{2k+3} - M\left(2^{k+2} + 3^{2k+1}\right)$	M1
	$= 2^{k+2}(2-M) + 3^{2k+1}(3^2 - M)$	A1
	$= (2-M)\left(2^{k+2} + 3^{2k+1}\right) + 3^{2k+1}\times 7$ or $(9-M)\left(2^{k+2} + 3^{2k+1}\right) - 7\times 2^{k+2}$ oe	dM1
	$\therefore \text{f}(k+1) = 2\text{f}(k) + 7\times 3^{2k+1}$ oe e.g. $9\text{f}(k) - 7\times 2^{k+2}$	A1
	Or e.g. $7\times 3^{2k+1}$ is a multiple of 7, so if f(k) is a multiple of 7 then f(k+1) is also a multiple of 7	
	If the result is true for $n = k$ it is also true for $n = k + 1$ (†)	
	As the result has been shown to be true for $n = 1$, it is true for all $n \in \mathbb{Z}^+$	A1 cso (6) **[12]**

9(i)	
B1	Check that the formula gives 1 when $n = 1$ Working must be shown. (Need not state true for $n = 1$ for this mark – but see final A)
M1	(Assume true for $n = k$ and) attempts to substitute the formula for u_k into $u_{k+1} = \frac{1}{3}(2u_k - 1)$ or equivalent with suffixes increased. Allow slips.
A1	Correct substitution.
dM1	Obtain an expression with $\left(\frac{2}{3}\right)^{k+1}$ and no other k. Alternatively, expands u_{k+1} to a matching expression (ie work from both directions).
A1	Correct expression when $n = k + 1$ At least one intermediate stage of working must be shown and no errors (though notational slips may be condoned). If working from both directions, it is for correct work to reach matching expressions.
A1cso	Correct concluding statements following correct solution which has included each of the points (†) at some stage during the working. Depends on all except the first B mark (e.g. if they think they have checked $n = 1$ but have really checked $n = 2$). Note: Allow the M's and first two A's for students who go from k+1 to k+2 but treat it as k to $k + 1$.
(ii)	
B1	Checks the case $n = 1$. Minimum statement of $f(1) = 35$
M1	Attempts an expression for $f(k + 1) - Mf(k)$ with any value of M. Need not be simplified. Most likely with $M = 1$ but may be seen with other values of M. With $M = 0$, $f(k+1) = 2^{k+3} + 3^{2k+3}$ is all that is required.
A1	A correct expression with terms 2^{k+2} and 3^{2k+1} clearly identified.
dM1	Attempts to extract/identify $f(k)$ within a correct expression to give terms divisible by 7. With $M = 0$ look for $f(k+1) = 2\times\left(2^{k+2} + 3^{3k+1}\right) + 7\times 3^{2k+1}$ or $9\times\left(2^{k+2} + 3^{3k+1}\right) - 7\times 2^{k+2}$ oe and similar for other value of M.
A1	One of the correct expressions for $f(k+1)$ shown (or with powers of 2 and 3) or full reason why $f(k+1)$ is divisible by 7, following a suitable expression.
A1cso	Correct concluding statements following correct solution which has included each of the points (†) at some stage during the working. Depends on all previous marks.

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