Question Number	Scheme	Notes	Marks
7	$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$		
7(a)	$n=1, \text{LHS}=1^2=1,\ \text{RHS}=\frac{1}{6}.2.3=1$	Shows both LHS=1 and RHS=1 Accept LHS = 1 but must see at least $\frac{1}{6}.2.3=1$ for RHS.	B1
	Assume true for $n=k$		
	When $n=k+1$ $\sum_{r=1}^{k+1} r^2 = \frac{k}{6}(k+1)(2k+1)+(k+1)^2$	Adds $(k+1)^2$ to result for $n=k$	M1
	$=\frac{(k+1)}{6}(k(2k+1)+6(k+1))$	Attempt to factorise by $\frac{(k+1)}{6}$	dM1
	$=\frac{(k+1)}{6}(k+2)(2k+3)$ $=\frac{(k+1)}{6}((k+1)+1))((2(k+1)+1)$	Either factorised form. SC allow dM1A0 for fully factorising to a cubic expression and going direct to the fully factorised expression with no intermediate quadratic seen.	A1
	True for $n=1$. If true for $n=k$ then true for $n=k+1$ therefore true for all n.	Complete proof with no errors and these 4 statements seen anywhere. Depends on both M's and the A, but may be scored if the B is lost as long as some indication of true for $n = 1$ is given.	A1cso
			(5)
7(b)	$\sum_{r=1}^{n}(r^2+2) = \sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} 2$ $=\frac{n}{6}(n+1)(2n+1)+\ldots$	Split into the addition of 2 sums and applies the result of (a).	M1
	$=\frac{n}{6}(n+1)(2n+1)+2n$	Correct expression.	A1
	$=\frac{n}{6}(2n^2+3n+13)$	Factorises out the $\frac{n}{6}$ - must have a common factor n to achieve this mark; Simplifies to correct answer.	M1; A1
	$(a=2, b=3, c=13)$		
			(4)
7(c)	$\sum_{r=10}^{25}(r^2+2) = S_{25}-S_9$ $=\frac{25}{6}.(2\times25^2+3\times25+13)-\frac{9}{6}(2\times9^2+3\times9+13)$	Attempts $S_{25}-S_9$ or $S_{25}-S_{10}$ with some substitution.	M1
	$=\frac{25}{6}\times1338-\frac{9}{6}\times202=5575-303=5272$	For 5272	A1
	Note: Answer only (from calculator) is M0A0 as question requires use of part (b).		
			(2)
			Total 11

Question Number	Scheme	Notes	Marks
	$f(n) = 4^{n+2} + 5^{2n+1}$ divisible by 21		
8	$n = 1, 4^3 + 5^3 = 189 = 9 \times 21$ (Or $n = 0,\ 4^2 + 5^1 = 21$)	$f(1) = 21 \times 9$ Accept f(0) = 21 as an alternative starting point.	B1
	Assume that for $n = k$, $f(k) = (4^{k+2} + 5^{2k+1})$ is divisible by 21 for $k \in \mathbb{Z}^+$.		
	$f(k+1) - f(k) = 4^{k+3} + 5^{2k+3} - (4^{k+2} + 5^{2k+1})$	Applies $f(k+1)$ with at least 1 power correct. May be just as f(k + 1), or as part of an expression in f(k + 1) and f(k).	M1
	$= 4.4^{k+2} + 25.5^{2k+1} - 4^{k+2} - 5^{2k+1}$	For a correct expression in f(k + 1), and possibly f(k), with powers reduced to those of f(k).	A1
	$= 3.4^{k+2} + 24.5^{2k+1}$		
	$= 3f(k) + 21.5^{2k+1}$ or $= 24f(k) - 21.4^{k+2}$	For one of these expression or equivalent with obvious factor of 21 in each.	A1
	$f(k+1) = 4f(k) + 21.5^{2k+1}$	Makes $f(k+1)$ the subject or gives clear reasoning of each term other than f(k+1) being divisible by 21. Dependent on at least one of the previous accuracy marks being awarded.	dM1
	{ $f(k+1)$ is divisible by 21 as both $f(k)$ and 21 are both divisible by 21}		
	If the result is **true for *n* = *k***, then it is now **true for *n* = *k* + 1**. As the result has shown to be **true for *n* = 1 (or 0)**, then the result is **true for all *n*** ($\in \mathbb{Z}^+$).	Correct conclusion seen at the end. Condone true for n = 1 stated earlier**.** Depends on both M's andA's, but may be scored if the B is lost as long as at least f(1) = 189 was reached (so e.g. if the 21×9 was not shown)	A1 cso
			(6)
ALT for first 4 marks	$n = 1, 4^3 + 5^3 = 189 = 9 \times 21$ (Or $n = 0,\ 4^2 + 5^1 = 21$)	As main scheme.	B1
	$f(k+1) - \alpha f(k) = 4^{k+3} + 5^{2k+3} - \alpha(4^{k+2} + 5^{2k+1})$	Attempts f(k + 1) in any equation (as main scheme).	M1
	$f(k+1) - \alpha f(k) = (4-\alpha)4^{k+2} + (25-\alpha)5^{2k+1}$	For a correct expression with any α, with powers reduced to match f(k).	A1
	$f(k+1) - \alpha f(k) = (4-\alpha)(4^{k+2} + 5^{2k+1}) + 21.5^{2k+1}$ $f(k+1) - \alpha f(k) = (25-\alpha)(4^{k+2} + 5^{2k+1}) - 21.4^{k+2}$	Any suitable equation with powers sorted appropriately to match f(k)	A1
	NB: $\alpha = 0, \alpha = 4, \alpha = 25$ will make relevant terms disappear, but marks should be awarded accordingly.		
			Total 6