Question	Scheme	Marks
9(a)	For $n = 1$, $\sum_{r=1}^{1} r^3 = 1$ and $\frac{1}{4}\left(1^2\right)\left(1+1\right)^2 = \frac{1}{4}\times 1\times 4 = 1$ So true for $n = 1$	**B1**
	(Assume the result is true for $n = k$, so $\sum_{r=1}^{k} r^3 = \frac{1}{4}k^2\left(k+1\right)^2$) Then $\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2\left(k+1\right)^2 + \left(k+1\right)^3$	**M1**
	$= \frac{1}{4}\left(k+1\right)^2\left[k^2+4(k+1)\right] = \frac{1}{4}\left(k+1\right)^2\left[k^2+4k+4\right]$	**M1**
	$= \frac{1}{4}\left(k+1\right)^2\left(k+2\right)^2$	**A1**
	$\left[= \frac{1}{4}\left(k+1\right)^2\left((k+1)+1\right)^2\right]$ Hence result is true for $n = k+1$. As **true for $n = 1$** and have shown **if true for $n = k$ then it is true for $n = k+1$, so it is true for all** $n \in \mathbb{N}$ by induction.	**A1**
		(5)
(b)	$\sum_{r=1}^{n} r\left(r+1\right)\left(r-1\right) = \sum_{r=1}^{n} r^3 - r$	**B1**
	$= \frac{1}{4}n^2\left(n+1\right)^2 - \frac{1}{2}n\left(n+1\right)$	**M1**
	(Please note the mark above is incorrectly labelled as A1 on e-PEN)	
	$= \frac{1}{4}n\left(n+1\right)\left[n^2+n-2\right] = \frac{1}{4}n\left(n+1\right)(n+...)(n+...)$	**M1**
	$= \frac{1}{4}n\left(n+1\right)\left(n-1\right)\left(n+2\right)$	**A1**
		(4)
(c)	$\sum_{r=n}^{2n} r^2 = \frac{1}{6}(2n)\left(2n+1\right)\left(2(2n)+1\right) - \frac{1}{6}(n-1)(n)\left(2(n-1)+1\right)$	**M1** **A1**
	$3\sum_{r=1}^{n} r\left(r+1\right)\left(r-1\right) = 17\sum_{r=n}^{2n} r^2$ $\Rightarrow \frac{3}{4}n\left(n+1\right)\left(n-1\right)\left(n+2\right) = \frac{17}{6}n\left(2(8n^2+6n+1)-(2n^2-3n+1)\right)$ $\Rightarrow 18\left(n+1\right)\left(n-1\right)\left(n+2\right) = 68\left(14n^2+15n+1\right) = 68\left(14n+1\right)(n+1)$	**dM1**

	$\Rightarrow 18(n-1)(n+2) = 68(14n+1)$ $\Rightarrow 18n^2 - 934n - 104 = 0 \Rightarrow n = ...$	**ddM1**
	$n = 52$	**A1**
		(5)
		(14 marks)

Notes:

(a)

B1: Checks the result for $n = 1$. Should see a clear substitution into both sides, accept minimum of seeing $\frac{1}{4}\times 1\times 4$, or $\frac{1}{4}\times 1\times 2^2$, or $\frac{1}{4}\times 1\times (1+1)^2 = 1$ for right hand side.

M1:(Makes or assumes the inductive assumption, and) adds $(k + 1)^3$ to the result for $n = k$

M1: Attempts to take at least $(k+1)^2$ as a factor out of the expression. Allow if an expansion to a quartic is followed by the factorised expression.

A1: Reaches the correct expression for $n = k + 1$from correct working with sufficient working seen, so expect at least seeing the quadratic before a factorised form.. Need not see the "k+1" explicitly for this mark.

A1: Completes the induction by demonstrating the result clearly, with suitable conclusion conveying "true for $n = 1$", "assumed true for $n = k$" and "shown true for $n = k + 1$", and "hence true for all n". All these statements (or equivalents) must be seen in their conclusion (not simply scattered through the work). Depends on all except the B mark, though a check for $n = 1$ must have been attempted.

(b)

B1: Correct expansion.

M1: Applies the standard formula for $\sum r$ and the result from (a) to their sum.

If the expansion is given as $\sum r^3 - r^2$, allow the use of $\sum r^2$ instead of $\sum r$

M1: Takes out the common factors n and $(n+1)$ and attempts to simplify to required form OR factorises their quartic.

A1: Correct answer. (Ignore A, B and C listed explicitly.) Correct answer can be obtained from a cubic or a quartic. Award M1A1 in either of these cases.

(c)

M1: Attempts to apply $\sum_{r=n}^{2n} r^2 = \sum_{r=1}^{2n} r^2 - \sum_{r=1}^{n-1} r^2$ with the standard result for $\sum r^2$ Accept with n instead of $n-1$ in second expression.

A1: Correct expression for the RHS seen, no need to be simplified.

dM1: Applies the summations to the equation in the question and cancels/factorises out the factor n. Depends on the first M mark of (c)

ddM1: Simplifies the quadratic factor of the right hand side and cancels/factors out the n+1 and solves the resulting quadratic. Note M1M1 is implied by sights of the correct roots $-\frac{1}{9}, 52, 0, -1$ of the quartic. Depends on both previous M marks in (c)

A1: Correct answer. Must reject other roots. The correct answer obtained from a quartic or cubic equation solved by calculator gains all relevant marks.

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