| Question | Scheme | Marks |
|---|---|---|
| **9** | For $n = 1$, $\sum_{r=1}^{1} \log(2r-1) = \log(2-1) = \log 1$ and $\log\left(\frac{(2\times 1)!}{2^1 1!}\right) = \log 1$<br>So true for $n = 1$ | **B1** |
| | (Assume the result is true for $n = k$, so $\sum_{r=1}^{k} \log(2r-1) = \log\left(\frac{(2k)!}{2^k k!}\right)$ Then)<br>$\sum_{r=1}^{k+1} \log(2r-1) = \log\left(\frac{(2k)!}{2^k k!}\right) + \log(2(k+1)-1)$ | **M1** |
| | $= \log\left(\frac{(2k)!}{2^k k!} \times (2k+1)\right)$ | **M1** |
| | $= \log\left(\frac{(2k+1)!}{2^k k!} \times \frac{2k+2}{2k+2}\right) = \log\left(\frac{(2k+2)!}{2^k \times 2(k+1)!}\right)$ | **M1** |
| | $= \log\left(\frac{(2k+2)!}{2^{k+1}(k+1)!}\right)$ | **A1** |
| | Hence result is true for $n = k+1$. As true for $n = 1$ and have shown if true for $n = k$ then it is true for $n = k+1$, so it is true for all $n \in \mathbb{N}$ by induction. | **A1** |
| | | **(6)** |
| | | **(6 marks)** |

**Notes:**

**(a)**

**B1:** Checks the result for $n = 1$. Must see **both** sides (possibly in one line) identified as log1 or 0 but may not see much more than this.

**M1:** Makes the inductive assumption (may be implied) and applies it to the question by adding the $(k+1)^{th}$ term to the expression for the sum to $k$ terms. Allow if there are minor slips (e.g. a missing factorial) if the intent is clear.

**M1:** Attempts to combine or split log terms appropriately. Not dependent, so may be scored if the wrong term is added in the previous M as long it is a log term.

**M1:** Introduces the relevant cancelling factors to achieve the $(2k + 2)!$ term. The introduction of the factors must shown or implied in an intermediate step. Alternatively, may decompose from the $k+1$ statement to achieve the same intermediate expression.

**A1:** Achieves correct expression from correct work (or correctly shows equivalence).

**A1:** Completes the induction by demonstrating the result clearly, with suitable conclusion conveying "true for $n = 1$", "assumed true for $n = k$" and "shown true for $n = k + 1$", and "hence true for all $n$". Depends on all except the B mark, though a check for $n = 1$ must have been attempted.

**NB** Allow the M's and first A if $n$ is used throughout but the steps are correct, but must have used a different variable for the final A.

Alt steps if splitting logs: $\displaystyle\sum_{r=1}^{k+1}\log(2r-1)=\log\left(\frac{(2k)!}{2^k k!}\right)+\log(2k+1)$ **M1**

$=\log(2k)!-\log(2^k k!)+\log(2k+1)$ **M1**

$=\log(2k+1)!-\log\left(\frac{2^{k+1}(k+1)!}{2\times(k+1)}\right)=\log\left(\frac{(2k+1)!(2k+2)}{2^k(k+1)!}\right)$ **M1**

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