Question Number	Scheme	Notes	Marks
8	$y^2 = 8x \quad P\left(2p^2, 4p\right) \quad Q\left(\frac{2}{p^2}, \frac{-4}{p}\right)$		
	Each part is marked separately. For example there is no credit in (c) for work seen in (b) unless that work is referred to in (c)		
(a) **Subs. both *x* and *y* into** $y^2 = 8x$	LHS or $y^2 \left\{= \left(\frac{-4}{p}\right)^2\right\} = \frac{16}{p^2}$ RHS or $8x \left\{= 8\times\frac{2}{p^2}\right\} = \frac{16}{p^2}$ So *Q* lies on the parabola* Allow e.g., $\left(\frac{-4}{p}\right)^2 = 8\left(\frac{2}{p^2}\right) \Rightarrow \frac{16}{p^2} = \frac{16}{p^2} \Rightarrow$ true	Substitutes both coordinates of *Q* into the parabola equation, obtains e.g., $\frac{16}{p^2}$ twice and makes minimal conclusion - e.g., shown/QED/true/proven/✓ Sight of just "$y^2 = 8x$" is insufficient but allow "$y_Q{}^2 = 8x_Q$"	B1*
			(1)
Alt **Subs. *x* or *y* to find *y* or *x***	$x = \frac{2}{p^2} \Rightarrow y^2 = 8\times\frac{2}{p^2}$ or $\frac{16}{p^2} \Rightarrow y = \frac{-4}{p}$ or $\pm\frac{4}{p}$ or $y = \frac{-4}{p} \Rightarrow \frac{16}{p^2} = 8x \Rightarrow x = \frac{2}{p^2}$ So *Q* lies on the parabola*	Substitutes one coordinate of *Q* into the parabola equation to correctly find the other coordinate and makes minimal conclusion - e.g., - e.g., shown/QED/true/proven/✓ Sight of just "$y^2 = 8x$" is insufficient but allow "$y_Q{}^2 = 8x_Q$"	B1*
			(1)
8(b)	Focus is (2, 0) or $x = 2$, $y = 0$ Could be seen on a diagram	Correct focus seen or used. Condone (0, 2) if $x = 2$, $y = 0$ used but award final A0	B1
	gradient of $PQ = \frac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}$ or $\frac{-\frac{4}{p}-4p}{\frac{2}{p^2}-2p^2}$ $\left\{= \frac{4p^3+4p}{2p^4-2} = \frac{2p^3+2p}{p^4-1} = \frac{2p\left(p^2+1\right)}{p^4-1} = \frac{2p}{p^2-1}\right\}$	Attempts the gradient of *PQ* condoning one term of incorrect sign. Allow this mark is they subsequently attempt to convert it to a normal gradient. Note that *m* may be obtained from $4p = 2mp^2 + c$, $-\frac{4}{p} = \frac{2m}{p^2} + c \Rightarrow m = ...$	M1
	e.g., $y - 4p = \frac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(x - 2p^2\right)$	Any correct equation for *PQ*. May use *Q*. Allow this mark to be implied if their equation would have been correct but errors were made **simplifying** a correct gradient.	A1
	If $y = mx + c$ is used, one of the following expressions oe for *c* must be reached following correct gradient seen: $c = 4p - 2p^2\left(\text{gradient}\right)$ or $c = \frac{-4}{p} - \frac{2}{p^2}\left(\text{gradient}\right)$		
	Examples with fully simplified gradient (see overleaf for a fuller list): $x = 2 \Rightarrow y - 4p = \frac{2p}{p^2-1}\left(2-2p^2\right) \Rightarrow y = \frac{4p-4p^3+4p^3-4p}{p^2-1} = 0$ or $y - 4p = \frac{2p}{p^2-1}\left(2-2p^2\right) \Rightarrow y - 4p = -4p \Rightarrow y = 0$ $y = 0 \Rightarrow -4p = \frac{2p}{p^2-1}\left(x-2p^2\right) \Rightarrow x = \frac{-4p^3+4p+4p^3}{2p} = 2$ $(2,0) \Rightarrow -4p = \frac{2p}{p^2-1}\left(2-2p^2\right) \Rightarrow -4p = -4p$ $\Rightarrow$ So *PQ* passes through the focus*	A1*

	Substitutes $x = 2$ and shows $y = 0$ or vice versa or substitutes both values and shows that the equation is true. Must have minimal conclusion e.g., shown/QED/true/proven/✓ and no incorrect work. Condone no conclusion if the mark in (a) was withheld for this reason only. The examples indicate the minimum **level** of algebra acceptable. With the exception of using (2, 0) with a fully simplified gradient, <u>look for substitution into the line followed by a further step which shows an expression that clearly leads to 0, 2 or e.g., $-4p$ or "1=1" followed by a minimal conclusion</u>		
	Work in "a" can only access the accuracy marks when $a = 2$ is substituted		**(4)**
Alt 1 **Grad *PF* = Grad *QF***	Focus is (2, 0) or $x = 2$, $y = 0$ Could be seen on a diagram	Correct focus seen or used. Condone (0, 2) if $x = 2$, $y = 0$ used but award final A0	B1
	gradient $PF = \frac{4p}{2p^2-2}$ or $\frac{-4p}{2-2p^2}$ and gradient $QF = \frac{\frac{4}{p}}{2-\frac{2}{p^2}}$ or $\frac{-\frac{4}{p}}{\frac{2}{p^2}-2}$	M1: Obtains expressions for both gradients condoning one term of incorrect sign in either or both expressions A1: Both correct expressions oe	M1 A1
	Grad $QF = \frac{4p}{2p^2-2}$ = Grad PF So PQ passes through the focus*	Shows that the gradients are the same plus minimal conclusion e.g., shown/QED/true/proven/✓ with no incorrect work. Condone no conclusion if penalised in (a).	A1*
	Note: A variation is to show grad *PF* or grad *QF* = grad *PQ* – marked as Alt		(**4**)
	Alt 2 Follows (similar triangles)		

8(b) <u>Examples</u> of minimum amount of algebra required with different expressions for gradient:

$$y-4p=\frac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(x-2p^2\right)$$

$x=2,\ y=...$	$x=2\Rightarrow y-4p=\frac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(2-2p^2\right)\Rightarrow y=\frac{8p+\frac{8}{p}-8p^3-8p+8p^3-\frac{8}{p}}{2p^2-\frac{2}{p^2}}=0$
$y=0,\ x=...$	$y=0\Rightarrow -4p=\frac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(x-2p^2\right)\Rightarrow x=\frac{-8p^3+\frac{8}{p}+8p^3+8p}{4p+\frac{4}{p}}=2$
$(2,\ 0)\Rightarrow$	$(2,0)\Rightarrow -4p=\frac{4p+\frac{4}{p}}{2p^2-\frac{2}{p^2}}\left(2-2p^2\right)\Rightarrow -4p=\frac{8p+\frac{8}{p}-8p^3-8p}{2p^2-\frac{2}{p^2}}\Rightarrow -4p=-4p$

$$y-4p=\frac{4p^3+4p}{2p^4-2}\left(x-2p^2\right)$$

$x=2,\ y=...$	$x=2\Rightarrow y-4p=\frac{4p^3+4p}{2p^4-2}\left(2-2p^2\right)\Rightarrow y=\frac{8p^3+8p-8p^5-8p^3+8p^5-8p}{2p^4-2}=0$ or $y-4p=\frac{4p^3+4p}{2p^4-2}\left(2-2p^2\right)\Rightarrow y=\frac{-4p^3-4p+4p^3+4p}{p^2+1}=0$
$y=0,\ x=...$	$y=0\Rightarrow -4p=\frac{4p^3+4p}{2p^4-2}\left(x-2p^2\right)\Rightarrow x=\frac{-8p^5+8p+8p^5+8p^3}{4p^3+4p}=2$

$(2, 0) \Rightarrow$	$(2,0) \Rightarrow -4p = \dfrac{4p^3+4p}{2p^4-2}\left(2-2p^2\right) \Rightarrow -4p = \dfrac{8p^3+8p-8p^5-8p^3}{2p^4-2} \Rightarrow -4p = -4p$	
$y - 4p = \dfrac{2p}{p^2-1}\left(x-2p^2\right)$		
$x = 2,\ y = ...$	$x = 2 \Rightarrow y - 4p = \dfrac{2p}{p^2-1}\left(2-2p^2\right) \Rightarrow y = \dfrac{4p-4p^3+4p^3-4p}{p^2-1} = 0$ or $y - 4p = \dfrac{2p}{p^2-1}\left(2-2p^2\right) \Rightarrow y - 4p = -4p \Rightarrow y = 0$	
$y = 0,\ x = ...$	$y = 0 \Rightarrow -4p = \dfrac{2p}{p^2-1}\left(x-2p^2\right) \Rightarrow x = \dfrac{-4p^3+4p+4p^3}{2p} = 2$	
$(2, 0) \Rightarrow$	$(2,0) \Rightarrow -4p = \dfrac{2p}{p^2-1}\left(2-2p^2\right) \Rightarrow -4p = -4p$	
Note that this not an exhaustive list (for example there are all the corresponding $y = mx + c$ approaches or those using Q) and the precise choice of algebra will vary widely but with the exception of the last example above this mark requires substitution into the line followed by a further step which shows an expression that clearly leads to 0, 2 or e.g., $-4p$ or "1=1" followed by a minimal conclusion (unless B0 was given in (a) for that reason).		
8(b) cont.	$y^2 = 8x \quad P\left(2p^2,\ 4p\right) \quad Q\left(\dfrac{2}{p^2},\ \dfrac{-4}{p}\right) \quad X\left(2,\ \dfrac{-4}{p}\right) Y\left(2p^2,\ \dfrac{-4}{p}\right)$	
Alt 2 **Similar triangles**	Focus is (2, 0) or $x = 2$, $y = 0$ Could be seen on a diagram	Correct focus seen or used. Condone (0, 2) if $x = 2$, $y = 0$ used but award final A0 — B1
	$\dfrac{XF}{XP} = \dfrac{\frac{4}{p}}{4p+\frac{4}{p}} \quad \dfrac{QX}{QY} = \dfrac{2-\frac{2}{p^2}}{2p^2-\frac{2}{p^2}}$	M1: Obtains expressions for two ratios condoning one term of incorrect sign in either or both expressions A1: Both correct expressions oe — M1 A1
	$\dfrac{XF}{XP} = \dfrac{1}{p^2+1} = \dfrac{QX}{QY}$ So Δs XFQ & YPQ are similar $\Rightarrow PQ$ passes through the focus*	Shows that the ratios are the same, makes reference to similarity plus minimal conclusion e.g., shown/QED/true/proven/✓ with no incorrect work. Condone no conclusion if penalised in (a). — A1*
		(4)
8 cont.	$y^2 = 8x \quad P\left(2p^2,\ 4p\right) \quad Q\left(\dfrac{2}{p^2},\ \dfrac{-4}{p}\right)$	
(c)	$y = \sqrt{8}x^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2}\sqrt{8}x^{-\frac{1}{2}}$ or $2y\dfrac{dy}{dx} = 8 \Rightarrow \dfrac{dy}{dx} = \dfrac{8}{2y}$ or $x = at^2,\ y = 2at \Rightarrow \dfrac{dx}{dt} = 2at,\ \dfrac{dy}{dt} = 2at \Rightarrow \dfrac{dy}{dx} = \dfrac{2a}{2at}$ (They may use p for t and/or $a = 2$)	Achieves an expression of the correct form (sign/coefficient errors only) for the gradient. There is no requirement for calculus so they may use e.g., $m_T = \dfrac{1}{t}$. Must go beyond $\dfrac{dx}{dy}$ — M1
	m_T at $P = \dfrac{1}{2}\sqrt{8}\dfrac{1}{\sqrt{2p^2}}$ or $\dfrac{8}{2\times 4p}$ or $\dfrac{2\times 2}{2\times 2\times p}$ $\left\{=\dfrac{1}{p}\right\}$ or m_T at $Q = -\dfrac{1}{2}\sqrt{8}\dfrac{1}{\sqrt{\frac{2}{p^2}}}$ or $\dfrac{8}{2\times\left(\frac{-4}{p}\right)}$ or $x = \dfrac{2}{p^2},\ y = \dfrac{-4}{p} \Rightarrow \dfrac{dx}{dp} = \dfrac{-4}{p^3},\ \dfrac{dy}{dp} = \dfrac{4}{p^2} \Rightarrow \dfrac{dy}{dx} = -p$ Any correct unsimplified expression for the tangent gradient in terms of p at either point. If awarding for Q via explicit differentiation must choose the negative root	A1

	Eqn of tgt at P: $y-4p=\frac{1}{p}\left(x-2p^2\right)$ oe or Eqn of tgt at Q: $y+\frac{4}{p}=-p\left(x-\frac{2}{p^2}\right)$ oe	M1: Correct straight line method for either point with their tangent gradient in terms of p (but allow if "a" also present) Coordinates correctly placed. If $y=mx+c$ is used must reach c = ... following correctly placed coordinates A1: Any correct unsimplified equation for either tangent	M1 A1
	Note: $y=mx+c$: At P, $y=\frac{1}{p}x+2p$ At Q, $y=-px-\frac{2}{p}$		
	$\frac{1}{p}\left(x-2p^2\right)+4p=-p\left(x-\frac{2}{p^2}\right)-\frac{4}{p}\Rightarrow x=...$ or $\frac{1}{p}x+2p=-px-\frac{2}{p}\Rightarrow x=...$ Eliminates y from their tangent equations and solves for x (See note below if eliminate x). Gradients must be different and no clear evidence of conversion of any line to a normal. Condone poor algebra.		M1
	$x\left(\frac{1}{p}+p\right)=-\frac{2}{p}-2p\Rightarrow x=\frac{-\left(2p^2+2\right)}{\left(p^2+1\right)}=-2$	$x=-2$ only	A1
	$y=\frac{1}{p}\left(-2-2p^2\right)+4p,\quad -p\left(-2-\frac{2}{p^2}\right)-\frac{4}{p},\quad \frac{1}{p}(-2)+2p,\quad -p(-2)-\frac{2}{p}$ **d**M1: Substitutes their x (a constant or function of p) into one of their two tangent equations to obtain an expression for y. **Requires previous M mark**.		**d**M1
	e.g., $y=2p-\frac{2}{p},\quad 2\left(p-\frac{1}{p}\right),\quad \frac{2}{p}\left(p^2-1\right),\quad \frac{2p^2-2}{p},\quad \frac{2(p+1)(p-1)}{p}$ A1: Correct y in simplest form – two terms which could be factorised in any correct way and/or written as a single fraction. Note there is no requirement for coordinate notation.		A1
	Note it is obviously possible to eliminate x. In this case, award the last 4 marks in this order: M1: Eliminates x and solves for y A1: Any correct y in simplest form **d**M1: Substitutes their y (a constant or function of p) into one of their two tangent equations to obtain an expression for x. **Requires previous M mark.** A1: $x=-2$		**(8)**
	Working which involves "a" where a is never replaced by 2 can score the Ms		**Total 13**