Question	Scheme	Marks
10.(i)	$n=1 \Rightarrow 3^{n-1}\begin{pmatrix}2n+3 & -n\\ 4n & 3-2n\end{pmatrix}=3^{0}\begin{pmatrix}2(1)+3 & -1\\ 4(1) & 3-2(1)\end{pmatrix}=\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}=\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^{1}$	**B1**
	Assume true for $n = k$ so that $\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^{k}=3^{k-1}\begin{pmatrix}2k+3 & -k\\ 4k & 3-2k\end{pmatrix}$	
	$\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^{k+1}=3^{k-1}\begin{pmatrix}2k+3 & -k\\ 4k & 3-2k\end{pmatrix}\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}$ or $\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}^{k+1}=3^{k-1}\begin{pmatrix}5 & -1\\ 4 & 1\end{pmatrix}\begin{pmatrix}2k+3 & -k\\ 4k & 3-2k\end{pmatrix}$	**M1**
	$=3^{k-1}\begin{pmatrix}10k+15-4k & -2k-3-k\\ 20k+12-8k & -4k+3-2k\end{pmatrix}$ or $3^{k-1}\begin{pmatrix}10k+15-4k & -5k-3+2k\\ 8k+12+4k & -4k+3-2k\end{pmatrix}$ or $3^{k-1}\begin{pmatrix}6k+15 & -3k-3\\ 12k+12 & -6k+3\end{pmatrix}$	**A1**
	$\left[=3^{k}\begin{pmatrix}2k+5 & -k-1\\ 4k+4 & -2k+1\end{pmatrix}\right]=3^{k}\begin{pmatrix}2(k+1)+3 & -(k+1)\\ 4(k+1) & 3-2(k+1)\end{pmatrix}$	**A1**
	If the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.	**A1cso**
		(5)
(ii)	$f(1)=8^{3}+6=518=74\times 7$ (so true for $n = 1$)	**B1**
	Assume true for $n = k$ so that $8^{2k+1}+6^{2k-1}$ is divisible by 7	
	$f(k+1)=8^{2k+3}+6^{2k+1}$	**M1**
	$=64\times\left(8^{2k+1}+6^{2k-1}\right)+\ldots$ or $36\times\left(8^{2k+1}+6^{2k-1}\right)+\ldots$	**dM1**
	$=64\times\left(8^{2k+1}+6^{2k-1}\right)-28\times 6^{2k-1}$ or $36\times\left(8^{2k+1}+6^{2k-1}\right)+28\times 8^{2k+1}$	**A1**
	So if the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.	**A1cso**
		(5)
(ii) Alt	$f(1)=8^{3}+6=518=74\times 7$ (so true for $n = 1$)	**B1**
	Assume true for $n = k$ so that $8^{2k+1}+6^{2k-1}$ is divisible by 7	
	$f(k+1)-Mf(k)=8^{2k+3}+6^{2k+1}-M\left(8^{2k+1}+6^{2k-1}\right)$	**M1**
	$=(64-M)\left(8^{2k+1}+6^{2k-1}\right)+\ldots$ or $(36-M)\left(8^{2k+1}+6^{2k-1}\right)+\ldots$	**dM1**
	$=(64-M)\left(8^{2k+1}+6^{2k-1}\right)-28\times 6^{2k-1}$ or $(36-M)\left(8^{2k+1}+6^{2k-1}\right)+28\times 8^{2k+1}$	**A1**
	$\Rightarrow f(k+1)-Mf(k)$ divisible by 7 $\Rightarrow f(k+1)$ divisible by 7. So if the result is true for $n = k$ then it is true for $n = k + 1$. As the result has been shown to be true for $n = 1$, then the result is true for all n.	**A1cso**
		(Total 10 marks)

Notes

(i)

B1: Shows the result is true for $n = 1$. The LHS may just be stated, for the RHS accept as a minimum either one unsimplified term or the 3^0 seen. (Conclusion not needed here if both sides have been found correctly.)

M1: Attempts $\begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}^{k+1}$ either way round using the result for $n = k$.

A1: Correct unsimplified matrix. The coefficients inside may be simplified but the common factor 3 not taken out directly.

A1: Achieves this result with no errors, via $3^{k-1}\begin{pmatrix} 6k+15 & -3k-3 \\ 12k+12 & -6k+3 \end{pmatrix}$ or $3^k\begin{pmatrix} 2k+5 & -k-1 \\ 4k+4 & -2k+1 \end{pmatrix}$ (oe with simplified linear terms).

A1cso: Suitable conclusion following fully correct work. Must include in some form the points "true for n =1", "true for $n = k$ implies true for $n = k + 1$" and conclude true for all n in the conclusion. Depends on the preceding MAA marks and at least stating the correct matrix for n = 1in the initial base case check (So B0M1A1A1A1 is possible).

(ii)

B1: Shows the result is true for $n = 1$, Must express as a multiple of 7 or clearly show the factor.

M1: Attempts f(k + 1)

dM1: Attempts to express f (k + 1) in terms of f (k). Note they may let f(k) = 7m where m is an integer and use this in the working.

A1: Correct expression for f (k + 1) in terms of f (k) (or m)

A1cso: Suitable conclusion following fully correct work. Must include in some form the points "true for n =1", "true for $n = k$ implies true for $n = k + 1$" and conclude true for all n in the conclusion. Depends on the preceding MdMA marks and finding at least f(1) = 518 (So B0M1dM1A1A1 is possible if all that is missing is showing the factor 7 in f(1)).

(ii) Alt

B1: Shows the result is true for $n = 1$. Must express as a multiple of 7 or clearly show the factor.

M1: Attempt f (k +1) – Mf (k) for any integer M. If $M = 0$ this is the main scheme. $M = 1$ may be seen frequently, but other value are possible.

dM1: Attempts to express f (k +1) – Mf (k) in terms of f (k) or otherwise show a common factor of 7.

A1: Correct expression for f (k +1) – Mf (k) in terms of f (k) or with clear common factor of 7 shown. Note if $M = 1$ is used, the expression becomes $63\times8^{2k+1}+35\times6^{2k-1}=7(9\times8^{2k+1}+5\times6^{2k-1})$ which is fine for dM1A1.

A1cso: Refers to divisibility of f (k +1) and makes suitable conclusion following fully correct work. Must include in some form the points "true for n =1", "true for $n = k$ implies true for $n = k + 1$" and conclude true for all n in the working. Depends on the preceding MdMA marks and finding at least f(1) = 518 (So B0M1dM1A1A1 is possible if all that is missing is showing the factor 7 in f(1)).

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