A1: Both coordinates correct and no incorrect ones (but ignore (27,6))

Question	Scheme	Marks
8.(a)	$\sum_{r=1}^{n} r\left(2r^2-3r-1\right)=\sum_{r=1}^{n}\left(2r^3-3r^2-r\right)$ $=2\times\frac{1}{4}n^2(n+1)^2-3\times\frac{1}{6}n(n+1)(2n+1)-\frac{1}{2}n(n+1)$	**M1** **A1**
	$\frac{1}{2}n^2(n+1)^2-\frac{1}{2}n(n+1)(2n+1)-\frac{1}{2}n(n+1)$ $=\frac{1}{2}n(n+1)\left[n(n+1)-(2n+1)-1\right]$	**M1**
	$=\frac{1}{2}n(n+1)\left[n^2-n-2\right]=\frac{1}{2}n(n+1)(n+1)(n-2)$ $=\frac{1}{2}n(n+1)^2(n-2)*$	**A1***
		(4)
(b)	$\sum_{r=n}^{2n} r\left(2r^2-3r-1\right)=\sum_{r=1}^{2n} r\left(2r^2-3r-1\right)-\sum_{r=1}^{n-1} r\left(2r^2-3r-1\right)$ $=\frac{1}{2}(2n)(2n+1)^2(2n-2)-\frac{1}{2}(n-1)(n)^2(n-3)$	**M1**
	$=\frac{1}{2}n(n-1)\left[4(2n+1)^2-n(n-3)\right]$	**M1**
	$=\frac{1}{2}n(n-1)\left[15n^2+19n+4\right]$	**A1**
	$=\frac{1}{2}n(n-1)(15n+4)(n+1)$	**A1**
		(4)
		(Total 8 marks)

Notes

(a) Note – attempts at induction score no marks.

M1: Expands the bracket and attempt to use at least one of the standard formulae correctly.

A1: Fully correct expression

M1: Attempts to factorise out at least $n(n+1)$ - both terms must have been common factors in the terms of their expression. If expanded to a quartic, there must be a clear attempt at factorisation in stages, directly to the given answer will be M0A0. (Note if they try to find roots there needs to be evidence that −1 is a repeated root before going direct to the given answer from these.)

A1*: cso Must have achieved a suitable correct intermediate stage with a quadratic in their working.

(b)

M1: Applies f(2*n*) – f(*k*) where *k* is *n* – 1 or *n* with the formula from (a) or allow from restarts using the standard formulae.

dM1: Attempts to factorise out $n(n-1)$ - which must be factors of their expression, so use of f(2*n*) −f(*n*) will score dM0. Accept for this mark if they expand from a correct expression and achieve the correct answer.

A1: Correct quadratic factor. May be implied if expansion to a quartic achieves the correct answer without intermediate factorisation shown.

A1: Correct expression

Question	Scheme	Marks
9.(a)	$\dfrac{3z-1}{2}=\dfrac{\lambda+5\mathrm{i}}{\lambda-4\mathrm{i}}\times\dfrac{\lambda+4\mathrm{i}}{\lambda+4\mathrm{i}}$	**M1**
	$=\dfrac{\lambda^2+9\lambda\mathrm{i}-20}{\lambda^2+16}$	**M1**
	$\dfrac{3z-1}{2}=\dfrac{\lambda^2+9\lambda\mathrm{i}-20}{\lambda^2+16}\Rightarrow z=\dfrac{2\left(\dfrac{\lambda^2+9\lambda\mathrm{i}-20}{\lambda^2+16}\right)+1}{3}$	**ddM1**
	$=\dfrac{\lambda^2-8}{\lambda^2+16}+\dfrac{6\lambda}{\lambda^2+16}\mathrm{i}$	**A1**
		(4)
(a) Way 2	$\dfrac{3z-1}{2}=\dfrac{\lambda+5\mathrm{i}}{\lambda-4\mathrm{i}}\Rightarrow 3z=\dfrac{2\lambda+10\mathrm{i}}{\lambda-4\mathrm{i}}+1=\dfrac{3\lambda+6\mathrm{i}}{\lambda-4\mathrm{i}}$ $\Rightarrow 3z=\dfrac{3\lambda+6\mathrm{i}}{\lambda-4\mathrm{i}}\times\dfrac{\lambda+4\mathrm{i}}{\lambda+4\mathrm{i}}$ or $z=\dfrac{\lambda+2\mathrm{i}}{\lambda-4\mathrm{i}}\times\dfrac{\lambda+4\mathrm{i}}{\lambda+4\mathrm{i}}$	**M1**
	$3z=\dfrac{3\lambda^2+18\lambda\mathrm{i}-24}{\lambda^2+16}$ or $z=\dfrac{\lambda^2+6\lambda\mathrm{i}-8}{\lambda^2+16}$	**M1**
	$3z=\dfrac{3\lambda^2+18\lambda\mathrm{i}-24}{\lambda^2+16}\Rightarrow z=...$	**ddM1**
	$=\dfrac{\lambda^2-8}{\lambda^2+16}+\dfrac{6\lambda}{\lambda^2+16}\mathrm{i}$	**A1**
(a) Way 3	$\dfrac{3z-1}{2}=\dfrac{\lambda+5\mathrm{i}}{\lambda-4\mathrm{i}}\Rightarrow(3x+3y\mathrm{i}-1)(\lambda-4\mathrm{i})=2\lambda+10\mathrm{i}$ $\Rightarrow 3\lambda x-\lambda+12y+(4+3\lambda y-12x)\mathrm{i}=2\lambda+10\mathrm{i}$	**M1**
	$\Rightarrow 3\lambda x+12y=3\lambda,\ 3\lambda y-12x=6$	**M1**
	$\Rightarrow x=...,y=...$	**ddM1**
	$z=\dfrac{\lambda^2-8}{\lambda^2+16}+\dfrac{6\lambda}{\lambda^2+16}\mathrm{i}$	**A1**
(b)	$\arg z=\dfrac{\pi}{4}\Rightarrow \mathrm{Re}\,z=\mathrm{Im}\,z\ (>0)\Rightarrow\lambda^2-6\lambda-8=0\Rightarrow\lambda=...$ or $\arg z=\dfrac{\pi}{4}\Rightarrow\dfrac{6\lambda}{\lambda^2-8}=\tan\dfrac{\pi}{4}=1\Rightarrow\lambda^2-6\lambda-8=0\Rightarrow\lambda=...$	**M1**
	(Also need $\mathrm{Re}(z),\mathrm{Im}(z)>0$, so $\lambda>0$) $\lambda=3+\sqrt{17}$	**A1**