A1: Correct quadratic factor. May be implied if expansion to a quartic achieves the correct answer without intermediate factorisation shown.

A1: Correct expression

Question	Scheme	Marks
9.(a)	$\dfrac{3z-1}{2}=\dfrac{\lambda+5\mathrm{i}}{\lambda-4\mathrm{i}}\times\dfrac{\lambda+4\mathrm{i}}{\lambda+4\mathrm{i}}$	**M1**
	$=\dfrac{\lambda^2+9\lambda\mathrm{i}-20}{\lambda^2+16}$	**M1**
	$\dfrac{3z-1}{2}=\dfrac{\lambda^2+9\lambda\mathrm{i}-20}{\lambda^2+16}\Rightarrow z=\dfrac{2\left(\dfrac{\lambda^2+9\lambda\mathrm{i}-20}{\lambda^2+16}\right)+1}{3}$	**ddM1**
	$=\dfrac{\lambda^2-8}{\lambda^2+16}+\dfrac{6\lambda}{\lambda^2+16}\mathrm{i}$	**A1**
		(4)
(a) Way 2	$\dfrac{3z-1}{2}=\dfrac{\lambda+5\mathrm{i}}{\lambda-4\mathrm{i}}\Rightarrow 3z=\dfrac{2\lambda+10\mathrm{i}}{\lambda-4\mathrm{i}}+1=\dfrac{3\lambda+6\mathrm{i}}{\lambda-4\mathrm{i}}$ $\Rightarrow 3z=\dfrac{3\lambda+6\mathrm{i}}{\lambda-4\mathrm{i}}\times\dfrac{\lambda+4\mathrm{i}}{\lambda+4\mathrm{i}}$ or $z=\dfrac{\lambda+2\mathrm{i}}{\lambda-4\mathrm{i}}\times\dfrac{\lambda+4\mathrm{i}}{\lambda+4\mathrm{i}}$	**M1**
	$3z=\dfrac{3\lambda^2+18\lambda\mathrm{i}-24}{\lambda^2+16}$ or $z=\dfrac{\lambda^2+6\lambda\mathrm{i}-8}{\lambda^2+16}$	**M1**
	$3z=\dfrac{3\lambda^2+18\lambda\mathrm{i}-24}{\lambda^2+16}\Rightarrow z=...$	**ddM1**
	$=\dfrac{\lambda^2-8}{\lambda^2+16}+\dfrac{6\lambda}{\lambda^2+16}\mathrm{i}$	**A1**
(a) Way 3	$\dfrac{3z-1}{2}=\dfrac{\lambda+5\mathrm{i}}{\lambda-4\mathrm{i}}\Rightarrow(3x+3y\mathrm{i}-1)(\lambda-4\mathrm{i})=2\lambda+10\mathrm{i}$ $\Rightarrow 3\lambda x-\lambda+12y+(4+3\lambda y-12x)\mathrm{i}=2\lambda+10\mathrm{i}$	**M1**
	$\Rightarrow 3\lambda x+12y=3\lambda,\ 3\lambda y-12x=6$	**M1**
	$\Rightarrow x=...,y=...$	**ddM1**
	$z=\dfrac{\lambda^2-8}{\lambda^2+16}+\dfrac{6\lambda}{\lambda^2+16}\mathrm{i}$	**A1**
(b)	$\arg z=\dfrac{\pi}{4}\Rightarrow \mathrm{Re}\,z=\mathrm{Im}\,z\ (>0)\Rightarrow\lambda^2-6\lambda-8=0\Rightarrow\lambda=...$ or $\arg z=\dfrac{\pi}{4}\Rightarrow\dfrac{6\lambda}{\lambda^2-8}=\tan\dfrac{\pi}{4}=1\Rightarrow\lambda^2-6\lambda-8=0\Rightarrow\lambda=...$	**M1**
	(Also need $\mathrm{Re}(z),\mathrm{Im}(z)>0$, so $\lambda>0$) $\lambda=3+\sqrt{17}$	**A1**

		(2)
		(Total 6 marks)

Notes

(a)

M1: Multiplies rhs by $\frac{\lambda+4\mathrm{i}}{\lambda+4\mathrm{i}}$

M1: Applies $\mathrm{i}^2 = -1$ in both numerator and denominator and obtains a real number in the denominator.

ddM1: Rearranges to $z = \ldots$

A1: Correct and in the required form, but accept $\frac{\lambda^2-8+6\lambda\mathrm{i}}{\lambda^2+16}$. Need not be fully simplified. Accept

e.g $\frac{3\lambda^2-24}{3\lambda^2+48}+\frac{18\lambda}{3\lambda^2+48}\mathrm{i}$

(a) Way 2

M1: Rearranges to $3z = \ldots$ (or $z = \ldots$) and multiplies numerator and denominator by the complex conjugate of their denominator.

M1: Applies $\mathrm{i}^2 = -1$ in both numerator and denominator and obtains a real number in the denominator.

ddM1: Rearranges to $z = \ldots$ if not already done so. If rearranged to z initially **M1ddM1** will be scored together.

A1: As per main scheme.

There may be variations on the rearrangement, but the key steps will remain the same.

(a) Way 3

M1: Cross multiplies, applies $z = x + \mathrm{i}y$, expands and applies $\mathrm{i}^2 = -1$ to achieve Cartesian form terms.

M1: Equates real and imaginary parts to form two equations with real coefficients.

ddM1: Solves the equations simultaneously to find x and y in terms of λ.

A1: As per main scheme.

(b)

M1: Sets the imaginary part of z equal to their real part of z, or divides these and sets equal to 1, and forms and solves the resulting quadratic in λ. (Need not be real roots for the M.)

Watch for answers to (a) with a negative imaginary component that do not consider the minus sign, as these should score M0 as they have not set real and imaginary parts equal.

A1: Correct exact answer only. The negative solution must have been rejected. Allow if both numerators were correct in (a) if there was a slip in the denominator only (e.g. λ^2+4 or $\lambda+16$) or if they were only out by a positive scale factor (e.g. lost the 3).