Question Number	Scheme	Notes	Marks
8	$f(k)=7^{k-1}+8^{2k+1}$		
	General guidance: Apply the Way that best fits the overall approach. Condone work in e.g., *n* instead of *k*. Allow use of –57 but if any different multiples of 57 are involved, e.g., 114, the last A1 additionally requires "114 is a multiple of/divisible by (but not "factor of") 57" oe for each case Ignore work re the divisibility of f(2), f(3) etc but starting with e.g., f(2) scores a max of 01110. **Final A1**: There must be evidence that $\text{true for } n=k \Rightarrow \text{true for } n=k+1$ but it could be minimal and be scored in a conclusion or a narrative or via both. So if e.g., "Assume true for $n=k$ ..." is seen in the work followed by "true for $n=k+1$" in a conclusion this is sufficient. May say "is divisible by 57" for "true". Condone "for all $n\in\mathbb{Z}$" but **not** $n\in\mathbb{R}$		
Way 1 $f(k+1)$ $-f(k)$	$n=1:\quad f(1)=\left[7^0+8^3=\right]513$, $513\div 57=9$ oe	Obtains 513 for f(1) and **shows** 513 is divisible by 57. Allow $\frac{1+512}{57}=9$	**B1**
	$\left[f(k+1)=\right]7^{(k+1)-1}+8^{2(k+1)+1}\left\{=7^k+8^{2k+3}\right\}$	Attempts $f(k+1)$	**M1**
	$\left[f(k+1)-f(k)=\right]$ $7\left(7^{k-1}\right)-7^{k-1}+8^2\left(8^{2k+1}\right)-8^{2k+1}$	Obtains expression for $f(k+1)-f(k)$ in 7^{k-1} and 8^{2k+1} only	**M1**
	$=6\left(7^{k-1}+8^{2k+1}\right)+57\left(8^{2k+1}\right)$ $\Rightarrow f(k+1)=7f(k)+57\left(8^{2k+1}\right)$ or $=63\left(7^{k-1}+8^{2k+1}\right)-57\left(7^{k-1}\right)$ $\Rightarrow f(k+1)=64f(k)-57\left(7^{k-1}\right)$	M1: Obtains expression for $f(k+1)$ in terms of $f(k)$. May not see $f(k+1)=...$ A1: Correct expression. **Must see** $f(k+1)=...$ Allow if e.g., $7f(k)$ written as $7\left(7^{k-1}+8^{2k+1}\right)$ or $7\left(7^{k-1}\right)+7\left(8^{2k+1}\right)$	**M1** **A1**
	Shown **true for *n* = 1** and if **true for *n* = *k* <u>then</u> true for *n* = *k* + 1** so **true for all *n*** $(\in\mathbb{Z}^+)$	Makes correct conclusion or narrative with no errors throughout. Minimum in **bold**. Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	**A1**
			(6)
Way 2 $f(k+1)=$	$n=1:\quad f(1)=\left[7^0+8^3=\right]513$, $513\div 57=9$ oe	Obtains 513 for f(1) and **shows** 513 is divisible by 57. Allow $\frac{1+512}{57}=9$	**B1**
	$\left[f(k+1)=\right]7^{(k+1)-1}+8^{2(k+1)+1}\left\{=7^k+8^{2k+3}\right\}$	Attempts $f(k+1)$	**M1**
	$\left[f(k+1)=\right]7\left(7^{k-1}\right)+8^2\left(8^{2k+1}\right)$	Obtains expression for $f(k+1)$ in 7^{k-1} and 8^{2k+1} only	**M1**
	$=7\left(7^{k-1}+8^{2k+1}\right)+57\left(8^{2k+1}\right)$ $\Rightarrow f(k+1)=7f(k)+57\left(8^{2k+1}\right)$ or $=64\left(7^{k-1}+8^{2k+1}\right)-57\left(7^{k-1}\right)$ $\Rightarrow f(k+1)=64f(k)-57\left(7^{k-1}\right)$	M1: Obtains expression for $f(k+1)$ in terms of $f(k)$. May not see $f(k+1)=...$ A1: Correct expression. **Must see** $f(k+1)=...$ Allow if e.g., $7f(k)$ written as $7\left(7^{k-1}+8^{2k+1}\right)$ or $7\left(7^{k-1}\right)+7\left(8^{2k+1}\right)$	**M1** **A1**
	Shown **true for *n* = 1** and if **true for *n* = *k* <u>then</u> true for *n* = *k* + 1** so **true for all *n*** $(\in\mathbb{Z}^+)$	Makes correct conclusion or narrative with no errors throughout. Minimum in **bold**. Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	**A1**
			(6)

Question Number	Scheme	Notes	Marks
8 **Way 3** $f(k+1)$ $-mf(k)$	$n=1:\quad f(1)=\left[7^0+8^3=\right]513,$ $513\div 57=9$ oe	Obtains 513 for f(1) and **shows** 513 is divisible by 57. Allow $\frac{1+512}{57}=9$	**B1**
	$\left[f(k+1)=\right]7^{(k+1)-1}+8^{2(k+1)+1}\left\{=7^k+8^{2k+3}\right\}$	Attempts $f(k+1)$	**M1**
	$f(k+1)-mf(k)$ $=7\left(7^{k-1}\right)-\left(7^{k-1}\right)m+8^2\left(8^{2k+1}\right)-\left(8^{2k+1}\right)m$	Obtains expression for $f(k+1)-mf(k)$ in 7^{k-1} and 8^{2k+1} only	**M1**
	e.g., $m=7\Rightarrow$ $f(k+1)-7f(k)=57\left(8^{2k+1}\right)$ $\Rightarrow f(k+1)=7f(k)+57\left(8^{2k+1}\right)$ e.g., $m=64\Rightarrow$ $f(k+1)-64f(k)=-57\left(7^{k-1}\right)$ $\Rightarrow f(k+1)=64f(k)-57\left(7^{k-1}\right)$	M1: Obtains expression for $f(k+1)$ in terms of $f(k)$ using a value for m. May not see $f(k+1)=...$ A1: A correct expression. **Must see** $f(k+1)=...$ Allow if $\beta f(k)$ written as $\beta\left(7^{k-1}+8^{2k+1}\right)$ or $\beta\left(7^{k-1}\right)+\beta\left(8^{2k+1}\right)$	**M1** **A1**
	Shown **true for *n* = 1** and if **true for *n* = *k* <u>then</u> true for *n* = *k* + 1** so **true for all *n*** $(\in\mathbb{Z}^+)$	Makes correct conclusion or narrative with no errors throughout. Minimum in **bold**. Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	**A1**
			(6)
Way 4 $f(k)=57\lambda$	$n=1:\quad f(1)=\left[7^0+8^3=\right]513,$ $513\div 57=9$ oe	Obtains 513 for f(1) and **shows** 513 is divisible by 57. Allow $\frac{1+512}{57}=9$	**B1**
	$\left[f(k+1)=\right]7^{(k+1)-1}+8^{2(k+1)+1}\left\{=7^k+8^{2k+3}\right\}$	Attempts $f(k+1)$	**M1**
	$\left[f(k+1)=\right]7\left(7^{k-1}\right)+8^2\left(8^{2k+1}\right)$	Obtains expression for $f(k+1)$ in 7^{k-1} and 8^{2k+1} only	**M1**
	$=7\left(7^{k-1}+8^{2k+1}\right)+57\left(8^{2k+1}\right)$ $f(k)=57\lambda\Rightarrow f(k+1)=399\lambda+57\left(8^{2k+1}\right)$ or $=7\times 57\lambda+57\left(8^{2k+1}\right)$ or $=64\left(7^{k-1}+8^{2k+1}\right)-57\left(7^{k-1}\right)$ $f(k)=57\lambda\Rightarrow f(k+1)=64\times 57\lambda-57\left(7^{k-1}\right)$ or $=3648\lambda-57\left(7^{k-1}\right)$	M1: Obtains expression for $f(k+1)$ in terms of λ with $f(k)=57\lambda$ seen. May not see $f(k+1)=...$ A1: Correct expression **Must see** $f(k+1)=...$	**M1** **A1**
	Shown **true for *n* = 1** and if **true for *n* = *k* <u>then</u> true for *n* = *k* + 1** so **true for all *n*** $(\in\mathbb{Z}^+)$	Makes correct conclusion or narrative with no errors throughout. Minimum in **bold**. Requires all previous marks but can follow B0 if that mark was withheld for omitting to show that 513 is divisible by 57.	**A1**
			(6)
			Total 6