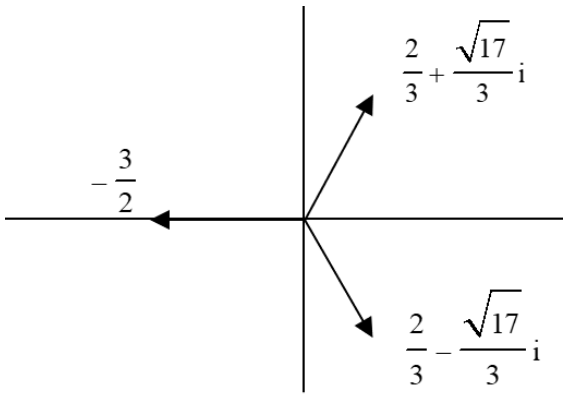
Question Number	Scheme	Notes	Marks
4	$\mathrm{f}(z)=6z^3+Az^2+Bz+C$	Condone work in x throughout	
(a)	$(z=)\dfrac{2}{3}-\dfrac{\sqrt{17}}{3}\mathrm{i}$	Correct conjugate. Must be seen in (a)	B1
			(1)
(b)	$\left(z-\left(\dfrac{2}{3}+\dfrac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z-\left(\dfrac{2}{3}-\dfrac{\sqrt{17}}{3}\mathrm{i}\right)\right)=\ldots\left\{z^2-\dfrac{4}{3}z+\dfrac{7}{3}\right\}$ or e.g. $\alpha+\beta=\dfrac{2}{3}+\dfrac{\sqrt{17}}{3}\mathrm{i}+\dfrac{2}{3}-\dfrac{\sqrt{17}}{3}\mathrm{i}=\ldots\left\{\dfrac{4}{3}\right\},\quad \alpha\beta=\left(\dfrac{2}{3}+\dfrac{\sqrt{17}}{3}\mathrm{i}\right)\left(\dfrac{2}{3}-\dfrac{\sqrt{17}}{3}\mathrm{i}\right)=\ldots\left\{\dfrac{4}{9}+\dfrac{17}{9}=\dfrac{7}{3}\right\}\Rightarrow\ldots\left\{z^2-\dfrac{4}{3}z+\dfrac{7}{3}\right\}$ M1: Completes a correct strategy for finding a quadratic factor and obtains a 3TQ with real coefficients. May be slips with the expansion or calculation/forming quadratic but score M0 if the starting point is clearly $\left(z+\left(\dfrac{2}{3}+\dfrac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z+\left(\dfrac{2}{3}-\dfrac{\sqrt{17}}{3}\mathrm{i}\right)\right)$ A1: Any correct three term quadratic factor. Allow any multiple of $z^2-\dfrac{4}{3}z+\dfrac{7}{3}$ e.g., $3z^2-4z+7$ May see $\left(3z-\left(2+\sqrt{17}\,\mathrm{i}\right)\right)\left(3z-\left(2-\sqrt{17}\,\mathrm{i}\right)\right)$ or $(3z-2)^2=\left(\pm\sqrt{17}\,\mathrm{i}\right)^2\Rightarrow\ldots\left\{9z^2-12z+21\right\}$		M1A1
	$6\left(z+\dfrac{3}{2}\right)\left(z^2-\dfrac{4}{3}z+\dfrac{7}{3}\right)=\ldots$ or e.g., $(2z+3)\left(3z^2-4z+7\right)=\ldots$ Multiplies their 3 term quadratic factor with real coefficients (or multiple) by $z+\dfrac{3}{2}$ (or multiple) to obtain a 4TC with real coefficients so allow $\left(z+\dfrac{3}{2}\right)\left(z^2-\dfrac{4}{3}z+\dfrac{7}{3}\right)=\ldots\left\{z^3+\dfrac{z^2}{6}+\dfrac{z}{3}+\dfrac{7}{2}\right\}$		M1
	$\left\{\mathrm{f}(z)=\right\}6z^3+z^2+2z+21$ or $A=1,\ B=2,\ C=21$	A1: Any two correct values for A, B or C (could be embedded) A1: Fully correct expression (ignore an "=0") or three correct values	A1A1
	Note that if the complex factors are not multiplied first e.g., $\left(z-\dfrac{3}{2}\right)\left(z-\left(\dfrac{2}{3}+\dfrac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z-\left(\dfrac{2}{3}-\dfrac{\sqrt{17}}{3}\mathrm{i}\right)\right)=\left(z^2+\left(\dfrac{5}{6}-\dfrac{\sqrt{17}}{3}\mathrm{i}\right)z-1-\dfrac{\sqrt{17}}{2}\mathrm{i}\right)\left(z-\left(\dfrac{2}{3}-\dfrac{\sqrt{17}}{3}\mathrm{i}\right)\right)$ $=z^3+\dfrac{1}{6}z^2+\dfrac{1}{3}z+\dfrac{7}{2}\Rightarrow 6z^3+z^2+2z+21$ Score the first 2 M marks together for obtaining a 4TC with real coefficients. M0M1 is possible e.g., with $\left(z+\dfrac{3}{2}\right)\left(z+\left(\dfrac{2}{3}+\dfrac{\sqrt{17}}{3}\mathrm{i}\right)\right)\left(z+\left(\dfrac{2}{3}-\dfrac{\sqrt{17}}{3}\mathrm{i}\right)\right)$ Any correct multiple of the 4TC scores the first A then score as main scheme		
			(5)

Question Number	Scheme/Notes	Marks
4(b)	**Alternative 1**: Substituting to obtain simultaneous equations $z=-\frac{3}{2}\Rightarrow -\frac{81}{4}+\frac{9}{4}A-\frac{3}{2}B+C=0$ $z=\frac{2}{3}\pm\frac{\sqrt{17}}{3}\text{i}\Rightarrow -\frac{188}{9}\mp\frac{10\sqrt{17}}{9}\text{i}+A\left(-\frac{13}{9}\pm\frac{4\sqrt{17}}{9}\text{i}\right)+B\left(\frac{2}{3}\pm\frac{\sqrt{17}}{3}\text{i}\right)+C=0$ $\Rightarrow -\frac{188}{9}-\frac{13}{9}A+\frac{2}{3}B+C=0,\quad \pm\left(-\frac{10\sqrt{17}}{9}+\frac{4\sqrt{17}}{9}A+\frac{\sqrt{17}}{3}B=0\right)$ M1: Substitutes $-\frac{3}{2}$ to obtain an equation and substitutes one of $\frac{2}{3}\pm\frac{\sqrt{17}}{3}$ and equates real and imaginary parts to obtain two further equations. All equations must have real coefficients and each variable must appear in at least one equation. A1: All three correct equations $13A-6B-9C=-188,\ 4A+3B=10,\ 9A-6B+4C=81$ $\Rightarrow A=1,\ B=2,\ C=21$ M1: Solves to obtain real values for A, B and C A1: Two correct values A1: All three correct values **Alternative 2**: Sum/product/pairwise product sum of roots of cubic $\text{sum}=\frac{2}{3}+\frac{\sqrt{17}}{3}\text{i}+\frac{2}{3}-\frac{\sqrt{17}}{3}\text{i}-\frac{3}{2}=\ldots\left\{-\frac{1}{6}\right\}=-\frac{A}{6}$ $\text{pairwise product sum}=-\frac{3}{2}\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\text{i}\right)-\frac{3}{2}\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\text{i}\right)+\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\text{i}\right)\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\text{i}\right)=\ldots\left\{\frac{1}{3}\right\}=\frac{B}{6}$ $\text{product}=-\frac{3}{2}\left(\frac{2}{3}+\frac{\sqrt{17}}{3}\text{i}\right)\left(\frac{2}{3}-\frac{\sqrt{17}}{3}\text{i}\right)=\ldots\left\{-\frac{7}{2}\right\}=-\frac{C}{6}$ M1: Obtains one equation in A, one in B and one in C all with real coefficients A1: All three correct equations $-\frac{1}{6}=-\frac{A}{6}\Rightarrow A=1,\quad \frac{1}{3}=\frac{B}{6}\Rightarrow B=2,\quad -\frac{7}{2}=-\frac{C}{6}\Rightarrow C=21$ M1: Solves to obtain real values for A, B and C Note that A, B and C would be implied by e.g., $-\frac{1}{6},\ \frac{1}{3},\ -\frac{7}{2}\Rightarrow x^3+\frac{1}{6}x^2+\frac{1}{3}x+\frac{7}{2}$ A1: Two correct values A1: All three correct values If real values for any of the sum/pairwise product sum/product are not explicitly seen allow the M marks if real values for A, B and C (which could be embedded) are obtained. The first A mark would then require all values correct (or a correct cubic) to be awarded. It is possible to e.g., find A and C as above and then use e.g., $\text{f}\left(-\frac{3}{2}\right)=0$ to determine B. In such cases the first M mark is scored when three equations have been attempted.	

4(b)	**Attempts that include long division:** If they find a quadratic factor as in the main scheme e.g., $z^2-\frac{4}{3}z+\frac{7}{3}$ allow M1A1 as before. Dividing it into f(z) can lead to equations $B-14+\frac{4}{3}A+\frac{32}{3}=0$ and $C-\frac{7}{3}\left(A+8\right)=0$ and these could be used with the equation $-\frac{81}{4}+\frac{9}{4}A-\frac{3}{2}B+C=0$ from using $\mathrm{f}\left(-\frac{3}{2}\right)=0$ or long division by $z+\frac{3}{2}$. Score the next M for obtaining real values for all constants and the A marks as usual. Other attempts including long division that do not find the quadratic factor as per the main scheme we will score as follows. Award the first M1 for credible work to obtain enough equations with real coefficients involving A, B and C so that values could be found, followed by A1 for correct equations. The next M1 is for solving to obtain values for A, B and C and then A1 for two correct values and final A1 for all 3 correct. If divided by $z+\frac{3}{2}$ the quadratic factor is $6z^2+\left(A-9\right)z+B-\frac{3}{2}A+\frac{27}{2}$ It is possible to apply the quadratic formula to this and equate the answer to $\frac{2}{3}\pm\frac{\sqrt{17}}{3}\mathrm{i}$ and generate further equations that way. (Long division by complex factors is unlikely but could lead to the other equations in Alt 1) There are potentially a lot of possible precise routes involving long division and coefficient comparison etc. but the above mark scheme principles apply.	

Question Number	Scheme/Notes	Marks
4(c)	 May use points or lines/vectors from origin. Note that B0B1 is not possible. 1st B1: real root plotted on negative x-axis and the conjugate pair plotted in Q1 and Q4, roughly aligned vertically. Ignore both axis scales and all labelling. 2nd B1: All roots plotted correctly and labelled. Only consider scale of x/real axis so both of the conjugate pair must be **clearly closer** to the y/imaginary axis than the real root. Award 2nd B0 if one of the conjugate pair is less than half the distance of the other from the x/real axis. If labelled with coordinates they must be the correct way around. Accept $\left(\frac{2}{3}, \pm\frac{\sqrt{17}}{3}\right)$ or $\left(\frac{2}{3}, \pm\frac{\sqrt{17}}{3}\mathrm{i}\right)$ if coordinates are used and accept these coordinates in vector notation or indicated by axis labels. If roots are only indicated by e.g. z_1, z_2, z_3 these must have been identified in (c) or earlier. 2nd B0 if axes labelled the wrong way round. (They do not need to be labelled or could be labelled x and y).	B1 B1
		(2)
		Total 8