Question Number	Scheme/Notes	Marks
8	Condone work in n instead of k throughout	
8(i)	$n=1\left\{\text{in}\begin{pmatrix}1-3n & 9n\\ -n & 3n+1\end{pmatrix}\right\}\Rightarrow\begin{pmatrix}1-3 & 9\\ -1 & 3+1\end{pmatrix}=\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ Obtains $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ or $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^1$ with (minimal) substitution seen. One of $1-3$ or $1-3(1)\to-2$ or $9(1)\to9$ or $-(1)\to-1$ or $3+1$ or $3(1)+1\to4$ is sufficient. No requirement to say "true" (oe) yet. Ignore further verifications for $n=2$ etc.	B1
	{Assume true for $n = k$:} $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^k=\begin{pmatrix}1-3k & 9k\\ -k & 3k+1\end{pmatrix}$	
	$\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^{k+1}=\begin{pmatrix}1-3k & 9k\\ -k & 3k+1\end{pmatrix}\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ or $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}\begin{pmatrix}1-3k & 9k\\ -k & 3k+1\end{pmatrix}$ Completes an attempt to form $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^{k+1}$ in terms of k	M1
	$=\begin{pmatrix}-2+6k-9k & 9-27k+36k\\ 2k-3k-1 & -9k+12k+4\end{pmatrix}$ or $\begin{pmatrix}-2+6k-9k & -18k+27k+9\\ -1+3k-4k & -9k+12k+4\end{pmatrix}$ or $\begin{pmatrix}-2-3k & 9+9k\\ -k-1 & 3k+4\end{pmatrix}$ Correct unsimplified or simplified matrix with no unexpanded expressions	A1
	$\left\{=\begin{pmatrix}-2-3k & 9+9k\\ -k-1 & 3k+4\end{pmatrix}\right\}=\begin{pmatrix}1-3(k+1) & 9(k+1)\\ -(k+1) & 3(k+1)+1\end{pmatrix}$ Reaches a correct matrix fully in terms of $k+1$ (terms in any order and allow for any $k + 1$ to be written as $1 + k$) with no errors. Meet in the middle approaches must be convincing. Requires previous two marks.	A1
	If the result is **true for $n = k$ then** it is **true for $n = k + 1$**. As the result has been shown to be **true for $n = 1$**, then the result is **true for** (all) n. Correct conclusion/narrative. All the elements in bold should be satisfied. Please consider the narrative and conclusion together. Allow poor phrasing if the intention is clear. "Assume $n = k$" in the narrative followed by "true for $n = k + 1$" in the conclusion plus "true for $n = 1$" and "true for (all) n" is sufficient. For the last statement allow "true for n", "true for $\square^+$", "true for $\square$" and condone "true for $\square$", "true for integers", "true for integers after 1" or similar but do not allow "true for all $n\in\square$" or just "true". Accept surrogates for "true" such as "correct for"/"it works for" etc. Requires previous 3 marks. Note that 01111 can only be awarded if the B mark was withheld for insufficient indication of substitution. If the base case work is omitted or wrong in any other way then 01110 is the maximum available.	A1
		(5)
	Note that is valid to e.g., assume true for $n=k+1$ and show true for $n=k+2$: $\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}^{k+2}=\begin{pmatrix}1-3(k+1) & 9(k+1)\\ -(k+1) & 3(k+1)+1\end{pmatrix}\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}=\begin{pmatrix}-3k-2 & 9k+9\\ -k-1 & 3k+4\end{pmatrix}\begin{pmatrix}-2 & 9\\ -1 & 4\end{pmatrix}$ $=\begin{pmatrix}6k+4-9k-9 & -27k-18+36k+36\\ 2k+2-3k-4 & -9k-9+12k+16\end{pmatrix}=\begin{pmatrix}-3k-5 & 9k+18\\ -k-2 & 3k+7\end{pmatrix}=\begin{pmatrix}1-3(k+2) & 9(k+2)\\ -(k+2) & 3(k+2)+1\end{pmatrix}$ Use Review for any such approaches you are not sure about.	

Question Number	Scheme/Notes	Marks
8(ii)	$u_1 = 1 \quad u_2 = 4 \quad u_{n+2} = 6u_{n+1} - 9u_n \quad n \in \mathbb{N} \Rightarrow u_n = 3^{n-2}(n+2)$	
	$n = 1 \Rightarrow u_1 = 3^{-1}(2+1) = 1, \quad n = 2 \Rightarrow u_2 = 3^0(2+2) = 4$ Obtains $u_1 = 1$ and $u_2 = 4$ from $u_n = 3^{n-2}(n+2)$ with some substitution seen for both cases although it may be minimal. Look for any numerical expressions that give 1 and 4. No requirement to say "true" (oe) yet. Ignore work for u_3 and beyond	B1
	{Assume true for $n = k$ and $n = k + 1$:} $u_k = 3^{k-2}(k+2)$ and $u_{k+1} = 3^{k-1}(k+3)$	
	$u_{k+2} = 6u_{k+1} - 9u_k = 6 \times 3^{k-1}(k+3) - 9 \times 3^{k-2}(k+2)$ Attempts u_k and u_{k+1} using $u_n = 3^{n-2}(n+2)$ and proceeds to attempt to use recurrence relation to obtain u_{k+2} in terms of k	M1
	$= 6 \times 3^k + 2k \times 3^k - 2 \times 3^k - k \times 3^k$ or e.g., $2 \times 3^k(k+3) - 3^k(k+2)$ Obtains an expression where all terms are multiples of 3^k. Requires previous mark	**d**M1
	$\{= 4 \times 3^k + k \times 3^k = 3^k(k+4)\} = 3^{(k+2)-2}((k+2)+2)$ or $3^{k+2-2}(k+2+2)$ Reaches a correct expression in terms of $k + 2$ with no errors. Meet in the middle approaches must be convincing.	A1
	If the result is **true for $n = k$ and $n = k + 1$ then** shown **true for $n = k + 2$**. As the result has been shown to be **true for $n = 1$ and $n = 2$**, then result is **true for** (all) ***n***. Correct conclusion/narrative. Please consider the narrative and conclusion together. Allow poor phrasing if the intention is clear. All the elements in bold should be satisfied. "Assume $n = k$ and $n = k + 1$" in the narrative followed by "true for $n = k + 2$" in the conclusion plus "true for $n = 1$ and $n = 2$" and "true for (all) n" or "true for $n \in \mathbb{N}^+$" is sufficient. For the last statement allow "true for n", "true for $\mathbb{N}^+$", "true for $\mathbb{N}$" and condone "true for $\mathbb{N}$", "true for integers", "true for integers after 1" etc. but do not allow "true for all $n \in \mathbb{N}$" or just "true". Accept surrogates for "true" such as "correct for"/"it works for" etc. Requires previous 3 marks. Note that 01111 can only be awarded if the B mark was withheld for insufficient indication of substitution. If just "$u_1 = 1, u_2 = 4$" is seen this is not sufficient evidence of any attempt to substitute and so the maximum score could only be 01110. The same applies if there are any errors in substitution. However e.g., just e.g., "when $n = 1$, $u_1 = 1$, when $n = 2$, $u_2 = 4$" can score 01111 since this implies an attempt to verify the values and no errors are seen.	A1
		(5)
		Total 10
	See overleaf for approaches that assume true for $n = k - 1$ and $n = k$ and show true for $n = k + 1$	

8(ii)	**Note that is valid to e.g., assume true for $n=k-1$ and $n=k$ and show true for $n=k+1$:** $n=1\Rightarrow u_1=3^{-1}(2+1)=1,\quad n=2\Rightarrow u_2=3^0(2+2)=4$ $u_{k-1}=3^{k-3}(k+1)\qquad u_k=3^{k-2}(k+2)$ $u_{k+1}=6u_k-9u_{k-1}=6\times3^{k-2}(k+2)-9\times3^{k-3}(k+1)$ $=2\times3^{k-1}(k+2)-3^{k-1}(k+1)$ or e.g., $2k\times3^{k-1}+4\times3^{k-1}-k\times3^{k-1}-3^{k-1}$ $=k\times3^{k-1}+3\times3^{k-1}=3^{k-1}(k+3)=3^{(k+1)-2}((k+1)+2)$ or $3^{k+1-2}(k+1+2)$ B1: As main scheme M1: Attempts u_{k-1} and u_k using $u_n=3^{n-2}(n+2)$ and proceeds to attempt to use recurrence relation to obtain u_{k+1} in terms of k dM1: Obtains an expression where all terms are multiples of 3^{k-1}. Requires previous mark A1: Reaches a correct expression in terms of $k+1$ with no errors. A1: If result is **true for $n=k-1$ and $n=k$ then** shown **true for $n=k+1$**. As the result has been shown to be **true for $n=1$ and $n=2$**, then result is **true for** (all) ***n***. See main scheme for guidance on the last mark. Use Review for any similar approaches you are not sure about.	