8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & 9n \\ -n & 3n+1 \end{pmatrix}$$

(5)

(ii) A sequence of numbers is defined by

$$u_1 = 1 \qquad u_2 = 4$$

$$u_{n+2} = 6u_{n+1} - 9u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3^{n-2}(n+2)$$

(5)

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(Total for Question 8 is 10 marks)

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