4: The rectangular hyperbola C has equation $xy = 81$

The point $P\left(9t, \frac{9}{t}\right)$, $t \neq 0$, lies on C.

(a) Use calculus to show that the normal to C at P has equation

$$ty = t^3x + 9\left(1 - t^4\right)$$

(4)

The normal to C at P meets the y-axis at the point A.

(b) Determine the exact coordinates of A in terms of t.

(2)

Given that $t = \frac{1}{3}$

(c) find the exact area of triangle OPA, where O is the origin.

(2)

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Question 4 continued

(Total for Question 4 is 8 marks)