6: $f(x) = 3x^2 + kx - 5$

where k is a constant.

The equation $f(x) = 0$ has roots α and β.

(a) State the value of $\alpha\beta$.

(1)

Given that $\alpha + \beta = 9\alpha\beta$

(b) determine the value of k.

(2)

(c) By first expanding $(\alpha + \beta)^3$ prove that

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

(2)

Without solving the equation $f(x) = 0$

(d) find a quadratic equation with integer coefficients that has roots

$$(\alpha^2 + \beta) \text{ and } (\alpha + \beta^2)$$

(6)

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Question 6 continued

(Total for Question 6 is 11 marks)