Question Number	Scheme	Notes	Marks
7	$f(z) = Pz^4 - 36z^3 + Qz^2 + 192z + 68, \quad z = 3 + 5i$ **Condone *x* used consistently for *z* throughout**		
(a)	$3 - 5i$	Correct second root	B1
			(1)
(b)	Either $(z-3-5i)(z-3+5i) = z^2 \pm 6z \pm m, \ m \in \mathbb{R}$ or $\alpha_1 + \alpha_2 = 6, \ \alpha_1\alpha_2 = M \Rightarrow z^2 \pm 6z \pm M, \ M \in \mathbb{R}$ For completing a correct strategy to find a 3 term quadratic factor. May either attempt to expand with correct starting point achieving $z^2 \pm 6z \pm m, \ m \in \mathbb{R}$, or attempts the sum and product of roots and reaches $z^2 \pm 6z \pm$ their (real) product		M1
	$z^2 - 6z + 34$	Correct factor. Accept answer only & "=0"	A1
			(2)
(c)	**Alternatives for (c) are shown overleaf**		
	$(z^2 - 6z + 34)("a"z^2 + "b"z + "c") = Pz^4 - 36z^3 + Qz^2 + 192z + 68$		
	$34c = 68 \Rightarrow c = 2$ "c" = 2 seen or implied. Allow $c = -2$ from $z^2 - 6z - 34$ but no other follow throughs.		B1 (ft on -34)
	$az^4 + (b-6a)z^3 + (c-6b+34a)z^2 + (34b-6c)z + 34c$ $\Rightarrow b - 6a = -36, \ c - 6b + 34a = Q, \ -6c + 34b = 192$ $\{a = P, \ 34c = 68\}$ With $c = 2$ substituted: $\Rightarrow b - 6a = -36, \ 2 - 6b + 34a = Q, \ -12 + 34b = 192$	Expands $("z^2 - 6z + 34")(az^2 + bz + c)$ [which could be implied] and compares coefficients for at least two of the z^3, z^2 and z terms obtaining at least 2 equations (could be implied) with real coefficients (may include the variable e.g., $34bz - 12z = 192z$). Must use at least 2 terms from the expansion per equation. Their 3TQ must have real coeffs.	M1
	e.g., $\Rightarrow 34b - 12 = 192 \Rightarrow b = 6, \quad b - 6a = -36 \Rightarrow a = 7 \Rightarrow P = 7$ Solves sufficient equations of correct form to find a real non-zero value for P (allow "a") or Q. No need to check algebra and accept a value following equations. Note that only 2 equations from comparing z^3 and z coefficients are needed to find P. It is possible to find Q first although it is not common e.g. $b = 6$ in $Q = 2 - 6b + 34\left(\frac{b}{6} + 6\right) \Rightarrow Q = 204$. **Previous mark required.**		**d**M1
	e.g., $Q = c - b + 34a = 2 - 36 + 238 = 204$ Solves sufficient equations of correct form to find real non-zero values for both P (allow "a") and Q. No need to check algebra and accept values following equations. **2 previous marks required.**		**dd**M1
	$P = 7$ and $Q = 204$ only Allow $a = 7$ if $P = a$ (only) seen	$P = 7$ (not "a") and $Q = 204$ No other answers. May be embedded in f(z)	A1
			(5)
(d)	"$7z^2 + 6z + 2" = 0$ (Allow with their other 3TQ factor - must have real coefficients) $\Rightarrow z = \frac{-6 \pm \sqrt{6^2 - 4 \times 7 \times 2}}{2 \times 7}$ or $\frac{-6 \pm \sqrt{-20}}{14}$ or $\frac{-6 \pm \sqrt{20}\,i}{14}$ or $z^2 + \frac{6}{7}z + \frac{2}{7} = \left(z + \frac{3}{7}\right)^2 - \frac{9}{49} + \frac{2}{7} = 0 \Rightarrow z = ... \left\{-\frac{3}{7} \pm \frac{\sqrt{-5}}{7}\right\}$ Either uses correct formula **correctly** or completes the square - usual rules - **shown** for solving their other three term quadratic factor (**which must have complex roots**). Do not accept factorisation or just writing down **simplified** roots from calculator. 1 root is sufficient. If forms equations e.g., $z_1 + z_2 = -\frac{6}{7}$, $z_1 z_2 = \frac{2}{7}$ (allow sign errors only) a **full** algebraic method for obtaining 1 root must be seen.		M1
	$\frac{-3 \pm i\sqrt{5}}{7}$ or $-\frac{3 \pm i\sqrt{5}}{7}$ or $\frac{-3}{7} \pm \frac{\sqrt{5}}{7}i$	Correct simplified other roots from correct factor. Must see "i". Ignore the presence of $3 \pm 5i$ if also listed. Ignore labelling. Does not require 5/5 in (c)	A1
			(2)
			Total 10

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7(c)	If long division is not completed/equations not seen allow access to the marks if the **correct** values for P and/or Q are deduced provided there is no clearly inappropriate work. For example it is possible to deduce that $b = 6$ as well as $c = 2$ and compare z^3 coeffs. from long division. If their quadratic factor is incorrect then equations must be seen for the 1st M1	
Alt 1a Full Long Division **Use Alt 1b if they have a <u>value</u> for c**	Allow for equivalent work using e.g., a multiplication grid $Pz^2+(6P-36)z+(Q+2P-216)$ $z^2-6z+34 \overline{)\, Pz^4-36z^3+Qz^2+192z+68}$ $\underline{Pz^4-6Pz^3+34Pz^2}$ $\underline{\underline{(6P-36)z^3+(Q-34P)z^2+192z}}$ $\underline{(6P-36)z^3-6(6P-36)z^2+34(6P-36)z}$ $(Q+2P-216)z^2+(1416-204P)z+68$ $\underline{(Q+2P-216)z^2-6(Q+2P-216)z+34(Q+2P-216)}$ $(120-192P+6Q)z-68P-34Q+7412$ $\Rightarrow 120-192P+6Q=0, -68P-34Q+7412=0$ B1**:** Attempts long division using a 3TQ with real coefficients and proceeds correctly to the first z^3 coefficient after subtraction (double underlined) - ignore subsequent errors. M1**:** Carries out sufficient long division and sets remainder = 0 (could be implied by equations if remainder not seen explicitly) to form a pair of linear simultaneous equations with real coefficients. Must have come from comparing coefficients oe for z terms and constant terms. Both equations must include P and Q.	B1 M1
	$\{32P-Q=20,\ 2P+Q=218\}$ $\Rightarrow P=...$ or $Q=...$ — Solves the equations to find a real non-zero value for either P or Q. No requirement to check algebra and accept a value following equations. **Previous mark required.**	**d**M1
	$\Rightarrow P=...$ and $Q=...$ — Solves the equations to find real non-zero values for both P and Q. No requirement to check algebra and accept values following equations. **2 previous marks required.**	**dd**M1
	$P = 7$ and $Q = 204$ only — $P = 7$ and $Q = 204$ No other answers. May be embedded in f(z)	A1
		(5)
Alt 1b Long Division: using a value for c	$Pz^2+(6P-36)z+2$ $z^2-6z+34 \overline{)\, Pz^4-36z^3+Qz^2+192z+68}$ $\underline{Pz^4-6Pz^3+34Pz^2}$ $(6P-36)z^3+(Q-34P)z^2+192z$ $\underline{(6P-36)z^3-6(6P-36)z^2+34(6P-36)z}$ $(Q+2P-216)z^2+(1416-204P)z+68$ $\underline{\qquad 2z^2 \qquad -12z+68}$ $(2P+Q-218)z^2-(204P-1428)z$ $\Rightarrow 2P+Q-218=0, -204P+1428=0$ B1: $c=2$ implied M1: Carries out sufficient long division and sets remainder = 0 (which could be implied) to form at least an equation with real coefficients in just P. Note that a remainder may not be seen explicitly. Must have come from comparing coefficients (oe) of z^2 terms and z terms. **d**M1: Solves an equation in P to find a real non-zero value for P **or** solves two equations (one in just P and one in both P and Q) to find a real non-zero value for Q. However, it is very unlikely that Q will be seen with no answer for P given. **dd**M1: Solves two equations to find real non-zero values for both P and Q. Must have had one equation in P only and one equation in both P and Q. A1: $P = 7$ and $Q = 204$ only. May be embedded in f(z)	

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7(c) Alt 2 Substitution	$f(3\pm5i)=P(3\pm5i)^4-36(3\pm5i)^3+Q(3\pm5i)^2+192(3\pm5i)+68$ $=P(-644\mp960i)+Q(-16\pm30i)+7772\pm600i$ or $-644P-16Q+7772+(\mp960P\pm30Q\pm600)i$ Correct **six term** expression for f(3+5i) or f(3−5i). May use calculator Implied by correct equations		B1
	$f(3\pm5i)=0\Rightarrow P(-644\mp960i)+Q(-16\pm30i)+7772\pm600i=0$ $\Rightarrow -644P-16Q+7772=0,\ \ \mp960P\pm30Q\pm600=0$ Attempts f(3+5i) or f(3−5i) and sets equal to 0 and equates real and imaginary parts to form a pair of linear simultaneous equations with real coefficients. Both equations must have no imaginary terms and both must include P and Q. Ignore extra equations		M1
	$\{161P+4Q=1943,\ \ 32P-Q=20\}$ $\Rightarrow P=...$ or $Q=...$	Solves the equations to find a real non-zero value for either P or Q. No requirement to check algebra and accept a value following equations. **Previous mark required.**	**d**M1
	$\Rightarrow P=...$ and $Q=...$	Solves the equations to find real non-zero values for both P and Q. No requirement to check algebra and accept values following equations. **2 previous marks required.**	**dd**M1
	P = 7 and Q = 204 only	P = 7 and Q = 204. No other answers. Maybe embedded in f(z)	A1
			(5)