Question Number	Scheme	Notes	Marks
8(a)	$\sum_{r=1}^{2n}(2r^2-1)=2\sum_{r=1}^{2n}r^2-\sum_{r=1}^{2n}1$ $=2\times\frac{2n}{6}(2n+1)\big(2(2n)+1\big)-\underline{\underline{2n}}$ $\left\{=\frac{2n}{3}(2n+1)(4n+1)-2n=\frac{16n^3}{3}+4n^2-\frac{4n}{3}\right\}$	M1: Uses $2n$ for k in $2\times\frac{k}{6}(k+1)(2k+1)$ Must replace k with $2n$ at least once. Must be otherwise correct i.e., only allow n or $2n$ for k Note that failing to replace the first k with $2n$ leads to $\frac{n}{3}(...)$ Award M0 if it is quite clear that the wrong formula for the sum of the squares has been used.	M1
		$\underline{\underline{\text{B1}}}$: Correct $\sum_{r=1}^{2n}1=2n$ seen/used	B1
	$=\frac{2n}{3}(8n^2+6n-2)$ or $\frac{4n}{3}(4n^2+3n-1)$ Obtains $...n\big($**3TQ** in $n\big)$ from a cubic with no constant. If 3TQ allow fractional coefficients. Allow implication of this mark with $\frac{4}{3}\big(4n^3+3n^2-n\big)$ or $\frac{2}{3}\big(8n^3+6n^2-2n\big)$ but not just $\frac{16n^3}{3}+4n^2-\frac{4n}{3}$. Must not have had a constant term. Can be scored if n instead of $2n$ used throughout the sum of squares formula $\left(\Rightarrow\frac{n}{3}\big(n^2+3n-5\big)\right)$ but next ddM0. Condone poor algebra.		M1
	$=\frac{4}{3}n(n+1)(4n-1)$ Allow e.g., $\frac{4n(4n-1)(n+1)}{3}$	Obtains $...n\big($factorised 3TQ in $n\big)$ Apply usual quadratic rules for the factorisation. If e.g., $\frac{2n}{3}(...)\rightarrow\frac{4n}{3}(...)$ work must be on a consistent 3TQ. Factors must have all real & exact terms. **Previous 2 method marks required.**	**dd**M1
		Correct result. Allow minor recovered algebraic/bracketing slips.	A1
	Note that e.g., if $2\times\frac{2n}{6}(2n+1)\big(2(2n)+1\big)-2n$ or $\frac{16n^3}{3}+4n^2-\frac{4n}{3}$ or $\frac{2n}{3}(2n+1)(4n+1)-2n$ is immediately followed by $\frac{4}{3}n(n+1)(4n-1)$ score 11000. Allow expanding the given answer $\frac{4}{3}n(n+1)(an+b)$ and equating coefficients for the last three marks: $\frac{16n^3}{3}+4n^2-\frac{4n}{3}=\frac{4}{3}an^3+\frac{4}{3}\big(a+b\big)n^2+\frac{4}{3}bn\Rightarrow a=4,\ b=-1\Rightarrow\frac{4}{3}n(n+1)(4n-1)$ M1: Correct form for expansion of given answer and obtains a value for either a or b **dd**M1: Obtains values for both a and b A1: $\frac{4}{3}n(n+1)(4n-1)$ (allow just $a=4,\ b=-1$ if $\frac{4}{3}n(n+1)(an+b)$ seen) Summation formulae must be used. No credit for attempts using induction or e.g., setting up simultaneous equations with 2 values of n, unless there is work that can score as above.		
			(5)

Question Number	Scheme	Notes	Marks
8(b)	$\sum_{r=1}^{1} r(3r-2)^2$ or LHS $= 1(3\times1-2)^2 = 1$ and $\frac{n^2(n+1)(9n-7)}{4}$ or RHS $= \frac{1^2\times2\times2}{4} = 1$ Achieves 1 from two numerical expressions (both not just "1"). No requirement here to say "true" etc. Allow as minimum, e.g., $1\times1 = \frac{4}{4} = 1$. "(When n = 1,) both =1" is B0 but final A1 available.		B1
	{Assume true for $n = k$, then} $\sum_{r=1}^{k+1} r(3r-2)^2 = \sum_{r=1}^{k} r(3r-2)^2 + (k+1)(3(k+1)-2)^2$ $= \frac{k^2(k+1)(9k-7)}{4} + (k+1)(3k+1)^2$	Adds an attempt at the $(k+1)$th term to an attempt at the sum to k terms. Allow clear copying slips (e.g., losing the squared from the k^2) but must be a recognisable attempt at forming $\frac{k^2(k+1)(9k-7)}{4} + (k+1)(3(k+1)-2)^2$ So e.g., $...+k(3k-2)^2$ is M0	M1
	$= \frac{(k+1)}{4}\left(9k^3-7k^2+4(9k^2+6k+1)\right) = \frac{(k+1)}{4}(9k^3+29k^2+24k+4)$ $\left\{\text{or } = (k+1)\left(\frac{9}{4}k^3+\frac{29}{4}k^2+6k+1\right)\right\}$ **OR** $= \frac{1}{4}\left(9k^4+38k^3+53k^2+28k+4\right)$ $\left\{\text{or } = \frac{9}{4}k^4+\frac{19}{2}k^3+\frac{53}{4}k^2+7k+1\right\}$ Reaches $\frac{(k+1)}{4}$(**4 term** cubic in k) or $(k+1)$(**4 term** cubic in k) **OR** expands to a **5 term** quartic. Must collect terms. Condone poor algebra. **Previous mark required**		**d**M1
	$= \frac{1}{4}(k+1)\left((k+1)(9k^2+20k+4)\right)$ or $\frac{1}{4}(k+1)^2(k+2)(9k+2) \Rightarrow \frac{(k+1)^2((k+1)+1)(9(k+1)-7)}{4}$ Correctly reaches the result completely in terms of $k + 1$ [but allow $k + 2$ or $k + 1 + 1$ for ($(k + 1) + 1$)] **with an intermediate step**. Condone poor notation and allow the odd recovered algebraic slip/poor bracketing provided they reach the correct $\frac{k+1}{4}$(4 term cubic in k) or $(k+1)$(4 term cubic in k) or 5 term quartic and there are no subsequent errors. Factorisation may be achieved via calculator use. Allow a final answer of $\frac{1}{4}(k+1)^2(k+2)(9(k+1)-7)$ and with brackets in any order. Meet in the middle approaches must clearly join and the final expression in $k + 1$ must be seen in the work		A1
	<u>True for $n = 1$</u> and <u>if true for $n = k$ then</u> it is also <u>true for $n = k + 1$</u> so by induction it is <u>true for (all) n</u> Acceptable proof (i.e., must have scored the previous A) and narrative/conclusion. **Requires previous three marks and can only follow B0 if B0 was given for <u>insufficient evidence</u> of substitution. There must be no errors if one substitution was attempted and must have reached both sides = 1.** "Assume (true) for $n = k$" or "If true for n = k" in narrative followed by "true for $n = k + 1$" is sufficient for the "if...then". Allow suitable surrogates for "true". Allow stating the result from the question paper or "P_n is true" <u>with added reference to n</u>. Condone $n \in \mathbb{Q}$ but not $n \in \mathbb{Q}$. Allow work in n rather than k throughout. There is no credit for attempts using summation formulae instead of induction unless there is work that can score via the scheme above		A1
			(5)

Question Number	Scheme	Notes	Marks
8(c)	$8\sum_{r=1}^{n} r(3r-2)^2 = 15\sum_{r=1}^{2n}(2r^2-1) \Rightarrow$ $8\times\frac{n^2(n+1)(9n-7)}{4} = 15\times\frac{4}{3}n(n+1)(4n-1)$ Attempts to form the given equation, obtaining an equation in n only with their answer to (a) which must be cubic but not necessarily in the right form. (Note - part (a) could have been reattempted in this part but do not allow attempts that seek to recreate an equivalent for the answer to (b) via summation formulae). Allow if 8 and 15 are swapped or if **one** is missing (but other correctly placed) provided there are no other errors. If 8 and 15 correctly placed, condone one minor copying error but must have quartic = cubic unless n instead of n^2 on LHS is the only error. Note that if they attempt to replace the 'n's in their answer to (a) with '$2n$'s then award M0 unless they have used n throughout part (a) in which case all marks are potentially available. If e.g., $15\times\frac{4}{3}$ is only seen evaluated it must be correct. Allow with "a" and "b" or invented values used but no further marks.		M1
	$\Rightarrow 2n(9n-7) = 20(4n-1)$ $\Rightarrow 18n^2 - 94n + 20 = 0$ $\Rightarrow 9n^2 - 47n + 10 = 0$ $\Rightarrow (9n-2)(n-5) = 0$ $\Rightarrow n = ...$ May see, e.g. $18n^3 - 94n^2 + 20n = 0$ $9n^3 - 47n^2 + 10n = 0$	Simplifies to a 3TQ or 3 term cubic with no constant (see below if 4TC or quartic) and solves to find a value for n. If working is shown, apply usual rules (in this case solution does not have to be a positive integer). However, if answer/s are just written down one real positive integer root must be achieved and be correct for their 3TQ/3TC with no constant. If there is no 3TQ/3TC with no constant and $n=5$ is just written down score 100. Do not allow solutions **directly** from just a quartic e.g., $9n^4 - 38n^3 - 37n^2 + 10n = 0$ or a cubic with constant e.g., $9n^3 - 38n^2 - 37n + 10 = 0$ **unless** there is a clear **full** method to factorise (e.g., factor theorem, long division, multiplication grid). Do not allow immediate factorisation. **Previous mark required.**	**d**M1
	$\{n=\}5$ Correct answer and no other unrejected solutions e.g., $n=\frac{2}{9}$, 0, -1 (Ignore any rejected incorrect values of n provided quadratic oe was correct and dM1 was scored) Answer may not be labelled and accept e.g., $x=5$		A1
			(3)
			Total 13