8: (a) Using the standard summation formulae show that

$$\sum_{r=1}^{2n}\left(2r^2-1\right)=\frac{4}{3}n(n+1)(an+b)$$

where a and b are integers to be determined.

(5)

(b) Prove by induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r(3r-2)^2 = \frac{n^2(n+1)(9n-7)}{4}$$

(5)

Using the results from parts (a) and (b) and showing all stages of your working,

(c) determine the value of n for which

$$8\sum_{r=1}^{n} r(3r-2)^2 = 15\sum_{r=1}^{2n}\left(2r^2-1\right)$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

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Question 8 continued

(Total for Question 8 is 13 marks)