Question Number	Scheme	Marks
9(a)	$y=\frac{c^2}{x}\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{c^2}{x^2}$ or $x\frac{\mathrm{d}y}{\mathrm{d}x}+y=0$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\frac{-ct^{-2}}{c}$	B1
	$y-\frac{c}{t}="-\frac{1}{t^2}"\left(x-ct\right)$ or $y="-\frac{1}{t^2}"x+b\Rightarrow \frac{c}{t}="-\frac{1}{t^2}"\left(ct\right)+b\Rightarrow b=...$	M1
	$t^2y-ct=-x+ct$ or $y=-\frac{1}{t^2}x+\frac{2c}{t}\Rightarrow t^2y=-x+2ct$ $\Rightarrow x+t^2y=2ct*$	A1*
		(3)
(b)	$y-\frac{c}{t}="t^2"\left(x-ct\right)$ or $y="t^2"x+b\Rightarrow \frac{c}{t}=ct^3+b\Rightarrow b=\ldots\Rightarrow y=t^2x+"\frac{c}{t}-ct^3"$	B1ft
		(1)
(c)	At $X, y=0\Rightarrow x=2ct$; At $Y, x=0\Rightarrow y=\frac{2c}{t}$	B1
	At Z, $x=0\Rightarrow y=\frac{c}{t}-ct^3$	M1
	E.g. Area $\Delta XYZ=\frac{1}{2}\left("\frac{2c}{t}"-\left("\frac{c}{t}-ct^3"\right)\right)\times"2ct"$	M1
	$=ct\left(\frac{c}{t}+ct^3\right)=c^2\left(1+t^4\right)*$	A1*
		(4)
(d)	$c^2\left(1+3^4\right)=410\Rightarrow c=\sqrt{5}\Rightarrow P\left(3\sqrt{5},\frac{\sqrt{5}}{3}\right)$	B1
	$PQ=2\times\sqrt{\left(3\sqrt{5}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2}$	M1
	$\frac{2\sqrt{410}}{3}$	A1
		(3)
		Total 11

Notes

(a)

B1: Any correct equation in $\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Correct straight line method with their tangent gradient in terms of t and/or c

A1: Achieves given answer with an intermediate step and no errors. Accept $t^2y + x = 2ct$

No calculus can score 011 max

(b)

B1ft: Correct equation with the negative reciprocal of their tangent gradient in terms of t and/or c. If $y = mx + b$ is used b must be correct for their t^2 and they must proceed to give the equation of the line.

(c)

B1: Correct x coordinate of X and correct y coordinate of Y. Allow if they mix up X and Y but the coordinates are correct.

M1: Uses $x = 0$ in their normal to find an expression for y at Z

M1: Correct full method for area of triangle XYZ with their coordinates. Condone for the M if they subtract y coordinates the wrong way and work with a "negative length" (but must treat the coordinate of Z correctly if combining separate triangles). Note that other methods are possible here, any correct full method is sufficient.

A1: Fully correct proof with intermediate step. Accept if the y coordinates were subtracted the wrong way but are later made positive.

Some alternative methods for the triangle area:

Using XY as base and PZ as height,

$$\text{Area } \Delta XYZ = \frac{1}{2}XY \times PZ = \frac{1}{2}\sqrt{(2ct)^2 + \left(\frac{2c}{t}\right)^2}\sqrt{(ct)^2 + \left(\frac{c}{t} - \frac{c}{t} + ct^3\right)^2}$$

$$= \frac{1}{2}2c\sqrt{t^2 + \frac{1}{t^2}}.c\sqrt{t^2 + t^6} = c^2\sqrt{\frac{t^4+1}{t^2}}\sqrt{t^2(t^4+1)} = c^2(t^4+1)$$

Shoelace approach,

$$\text{Area } \Delta XYZ = \frac{1}{2}\begin{vmatrix} 2ct & 0 & 0 \\ 0 & \frac{2c}{t} & \frac{c}{t} - ct^3 \end{vmatrix} = \frac{1}{2}\left(2ct \times \frac{2c}{t} - 2ct\left(\frac{c}{t} - ct^3\right)\right) = \frac{1}{2}(4c^2 - 2c^2 + 2ct^4) = c^2(1 + t^4)$$

Separate triangles,

$\text{Area } \Delta XYZ = \frac{1}{2}y_Y \times x_X - \frac{1}{2}y_Z \times x_X$ follows main scheme once factorised.

Other combinations are possible.

(d)

B1: Correct coordinates of P

M1: Correct attempt at PQ for their P or any other full method. May find Q and then distance PQ directly for example. They may identify Q by symmetry or via a longer approach finding the equation of the line through O and P and then finding where this intersects the hyperbola again.

A1: Correct exact value