9. The rectangular hyperbola H has equation $xy = c^2$ where $c > 0$

The point P lies on H and has coordinates $\left(ct, \frac{c}{t} \right)$ where $t > 0$

The line l_1 is the tangent to H at P

(a) Use calculus to show that an equation for l_1 is

$$x + t^2 y = 2ct$$

(3)

The line l_2 is the normal to H at P

(b) Determine an equation for l_2

(1)

Given that

- l_1 crosses the x-axis at the point X and crosses the y-axis at the point Y
- l_2 crosses the y-axis at the point Z

(c) show that the area of the triangle XYZ is given by

$$c^2\left(1 + t^4\right)$$

(4)

When $t = 3$

- the area of the triangle XYZ is 410
- the line from P through the origin meets H again at the point Q

(d) Determine the exact length of PQ, giving your answer as a simplified surd.

(3)

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Question 9 continued

(Total for Question 9 is 11 marks)