Question Number	Scheme	Marks
	Condone working in other letters such as *n* etc	
10	If $n=1$, $\sum_{r=1}^{n} r^2(2r-1)=1$ and $\frac{1}{6}n(n+1)(3n^2+n-1)=1$, **(**LHS=RHS**) (**so **true for *n* = 1)**.	B1
	$\sum_{r=1}^{k+1} r^2(2r-1)=\frac{1}{6}k(k+1)(3k^2+k-1)+(k+1)^2(2(k+1)-1)$	M1
	$=\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$ **or** $=\frac{1}{6}(k^2+3k+2)(3k^2+7k+3)$	A1
	$=\frac{1}{6}(k+1)(k+2)(3k^2+7k+3)=\frac{1}{6}(k+1)((k+1)+1)(3(k+1)^2+(k+1)-1)$	A1
	True for *n* = *k* + 1 if true for *n* = *k*, and as true for *n* = 1 true (by induction**) for all *n*.** (true for n = 1 may be stated at the start and not at the end which is fine)	A1cso
		(5)
		(Total 5)

Notes

The conclusion may be given as a narrative throughout the question, or it may be stated at the end, but the statements in bold are required somewhere in the solution

In all working condone the occasional missing bracket etc if recovered

B1: Shows true for $n = 1$. Don't need to see the substitution explicitly, but need to see that both sides have been evaluated when $n = 1$ and arrive at 1 for both sides- getting 1 for both sides is sufficient. Don't need to explicitly say LHS = RHS but must show clearly that both sides are equivalent by evidence of evaluating both.

M1: Adds the $(k + 1)^{th}$ term to the sum of the first k terms.

A1: Obtains $\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$ **or** $=\frac{1}{6}(k^2+3k+2)(3k^2+7k+3)$ from correct work (i.e. extracts 'first' linear factor or expresses as the product of two quadratics, and allow one term to be unsimplified after the 1/6 has been extracted).

A1: Achieves this result with no errors and $3k^2+7k+3$ seen (i.e. extracts 'second' linear factor and progresses to showing the correct form required with $n = k + 1$). Condone '$k + 2$' in place of '$(k + 1) + 1$

Allow work that shows equivalence between

e.g. $\frac{1}{6}(k+1)(3k^3+13k^2+17k+6)$ and $\frac{1}{6}(k+1)(k+2)(3(k+1)^2+(k+1)-1)$

A1: Full conclusion and all previous marks scored. As long as the conclusion is conveyed throughout the solution then this mark can be scored if all previous marks are scored.

For the final mark

True for $n=k \Rightarrow$ true for $n=k+1$, and as/if true for $n=1$, true for all $n \in \mathbb{N}$ $\left(\text{or } n \in \mathbb{Z}^+ \text{ or integers}\right)$

Or

True for $n=k+1$ when true for $n = k$ and as/if true for $n=1$, true for all $n \in \mathbb{N}$ $\left(\text{or } n \in \mathbb{Z}^+ \text{ or integers}\right)$

Condone 'true for all $n \in \mathbb{Z}$' but do not allow 'true for $n \in \mathbb{R}$' or just 'true'

Accept ‘correct/it works for’ in place of ‘true’

Note that some candidates may resort to ‘assume true for $n = k + 1$ and show true for $n = k + 2$ etc which marks the same way as above

M1: $\sum_{r=1}^{k+2} r^2(2r-1) = \frac{1}{6}(k+1)\left(k+2\right)(3\left(k+1\right)^2 + \left(k+1\right)-1) + (k+2)^2(2(k+2)-1)$

A1: $= \frac{1}{6}(k+2)(3k^3 + 22k^2 + 52k + 39)$

A1: $= \frac{1}{6}(k+2)(k+3)(3k^2 + 13k + 13) = \frac{1}{6}(k+2)(k+3)(3\left(k+2\right)^2 + \left(k+2\right)-1)$

A1: Full conclusion and all previous marks scored.

If there are any attempts you are not sure about, please send to review
Any attempts which do not use proof by induction score 0 marks