Question Number	Scheme	Notes	Marks
5.	$x^2 - 2x + 3 = 0$		
(a)(i)	$\alpha + \beta = 2, \quad \alpha\beta = 3$		B1
(ii)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4 - 6 = -2$		M1A1
(iii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = 8 - 3\times 3\times 2 = -10$		M1A1
			(5)
(b)(i)	$(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = \alpha^4 + 2(\alpha\beta)^2 + \beta^4 - 2(\alpha\beta)^2 = \alpha^4 + \beta^4$		B1*
ALT 1	$\alpha^4 + \beta^4 = \alpha^4 + \beta^4 + 2\alpha^2\beta^2 - 2\alpha^2\beta^2 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$		B1*
ALT 2	$(\alpha + \beta)^4 = \alpha^4 + 4\alpha^3\beta + 6\alpha^2\beta^2 + 4\alpha\beta^3 + \beta^4$ $\Rightarrow \alpha^4 + \beta^4 = (\alpha + \beta)^4 - 4\alpha\beta(\alpha^2 + \beta^2) - 6\alpha^2\beta^2$ $= (\alpha + \beta)^4 - 4\alpha\beta[(\alpha + \beta)^2 - 2\alpha\beta] - 6\alpha^2\beta^2$ $= (\alpha + \beta)^4 - 4\alpha\beta(\alpha + \beta)^2 + 2\alpha^2\beta^2$ $= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 4\alpha^2\beta^2 + 2\alpha^2\beta^2$ $= (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$		B1*
(ii)	$\text{Sum} = \alpha^3 + \beta^3 - (\alpha + \beta) = -10 - 2 = -12$		B1
	$\text{product} = (\alpha^3 - \beta)(\beta^3 - \alpha) = (\alpha\beta)^3 - (\alpha^4 + \beta^4) + \alpha\beta = 27 - (4 - 18) + 3 = 44$		M1A1
	$\Rightarrow x^2 + 12x + 44 = 0$		M1A1
			(6)
			Total 11

Notes

(a)(i)

B1: Both $\alpha + \beta = 2, \quad \alpha\beta = 3$

(ii)

M1: Use of a correct identity for $\alpha^2 + \beta^2$ (May be implied by their work) e.g. $(\text{their sum})^2 - 2\times \text{their product}$

A1: -2 from a correct solution only.

(iii)

M1: Use of a correct identity for $\alpha^3 + \beta^3$ (May be implied by their work) e.g.

$(\text{their sum})^3 - 3\times \text{their product} \times \text{their sum}$

A1: -10 **from a correct solution only.**

(b)(i)

B1*: Correct algebraic proof starting with RHS. Must see full correct expansion of $\left(\alpha^2+\beta^2\right)^2$ before the printed answer and no incorrect work.

ALT 1 Starts with LHS but must see $\alpha^4+\beta^4=\alpha^4+\beta^4+2\alpha^2\beta^2-2\alpha^2\beta^2$ before the printed answer and no incorrect work

ALT 2 Starts with the expansion of $\left(\alpha+\beta\right)^4$ then rearranges to obtain $\alpha^4+\beta^4$, factorises and progresses to the correct printed answer with no errors

In all attempts accept $2\alpha^2\beta^2$ as an equivalent to $2\left(\alpha\beta\right)^2$

(ii)

NOTE: there are no ft marks available in part (b)

B1: Correct working without using explicit roots leading to a correct sum.
M1: Attempts to expand. Must have at least three correct terms to score this mark.
A1: Correct product.
M1: Applies $x^2-\left(\text{sum}\right)x+\text{product}$ (may have to check for their sum and product and = 0 not required for this mark)
A1: $x^2+12x+44=0$ (must have the = 0 for this mark)
If p, q, r, stated but quadratic not stated then M1A0

NOTE: any attempts which try to solve the equation to find the roots will generally score zero