4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Given that $z = x + iy$, where x and y are real numbers, solve the equation

$$(z - 2i)(z^* - 2i) = 21 - 12i$$

where z^* is the complex conjugate of z.

(6)

(Total for Question 4 is 6 marks)

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5. The quadratic equation

$$x^2 - 2x + 3 = 0$$

has roots α and β.

Without solving the equation,

(a) (i) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

(ii) show that $\alpha^2 + \beta^2 = -2$

(iii) find the value of $\alpha^3 + \beta^3$

(5)

(b) (i) show that $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$

(ii) find a quadratic equation which has roots

$$(\alpha^3 - \beta) \text{ and } (\beta^3 - \alpha)$$

giving your answer in the form $px^2 + qx + r = 0$ where p, q and r are integers.

(6)

(Total for Question 5 is 11 marks)
