| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **5** | $2x^2 - x + 3 = 0$<br>Note that solving the equation and clearly using the values $\alpha = \frac{1+\sqrt{23}i}{4}$ and $\beta = \frac{1-\sqrt{23}i}{4}$<br>throughout i.e. for $\alpha^3+\beta^3$, $\alpha^4\beta-2+\alpha\beta^4-2$ & $\left(\alpha^4\beta-2\right)\left(\alpha\beta^4-2\right)$ is likely to only be able to access the final M mark in (c). However, if identities are used in (b) and (c) allow all the marks to be accessible. | | |
| **(a)** | $\alpha+\beta=\frac{1}{2},\quad \alpha\beta=\frac{3}{2}$ | Both values correct. B0 if these have clearly come from solving the equation. | **B1** |
| | | | **(1)** |
| **(b)** | $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=\left(\frac{1}{2}\right)^3-3\times\frac{3}{2}\times\frac{1}{2}$<br>Obtains a numerical expression for $\alpha^3+\beta^3$ with a **correct** identity. Condone substitution slips e.g., mixing up "$\frac{1}{2}$" and "$\frac{3}{2}$" and allow M1 if a correct identity is seen or implied. Sight of a correct identity followed by a value is sufficient but if only a value is seen it must be $-\frac{17}{8}$ | | **M1** |
| | $=-\frac{17}{8}$ oe, $-2.125$ | Correct value. Must not be fortuitous but allow all part (c) marks. | **A1** |
| | | | **(2)** |
| **(c)** | Work for the new sum/product may be embedded within a quadratic | | |
| | new sum $=\left\{\alpha^4\beta-2+\alpha\beta^4-2=\right\}\alpha\beta\left(\alpha^3+\beta^3\right)-4=\frac{3}{2}\left(-\frac{17}{8}\right)-4\left\{=-\frac{115}{16}\right\}$<br>Obtains a numerical expression for the new sum with a **correct** identity. Condone substitution slips e.g., mixing up "$\frac{1}{2}$" and "$\frac{3}{2}$" and allow M1 if a correct identity is seen or implied. Also ft their$\alpha^3+\beta^3$ which may be recalculated wrongly and may be 0. Sight of a correct identity followed by a value is sufficient but if only a value is given it must be $-\frac{115}{16}$ | | **1st M1 (sum)** |
| | new pro_duct $=\left\{\left(\alpha^4\beta-2\right)\left(\alpha\beta^4-2\right)=\right\}(\alpha\beta)^2-2\alpha\beta\left(\alpha^3+\beta^3\right)+4=\left(\frac{3}{2}\right)^2-2\times\left(\frac{1}{2}\right)^3\left(-\frac{17}{8}\right)+4\left\{=\frac{575}{32}\right\}$<br>Obtains a numerical expression for the new product with a **correct** identity. Condone substitution slips e.g., mixing up "$\frac{1}{2}$" and "$\frac{3}{2}$" and allow M1 if a correct identity is seen or implied. Also ft their$\alpha^3+\beta^3$ which may be recalculated wrongly and may be 0. Sight of a correct identity followed by a value is sufficient but if only a value is given it must be $\frac{575}{32}$ | | **2nd M1 (prod.)** |
| | new sum $=-\frac{115}{16}$ **or** new product $=\frac{575}{32}$ | One correct value. Allow -7.1875, 17.96875<br>**Requires one previous M mark.** | **A1 (M1 on ePen)** |
| | $x^2-\left(-\frac{115}{16}\right)x+\frac{575}{32}(=0)$<br>Correctly applies $x^2$ – (their new sum)$x$ + their new product. If e.g., the "$x$" is missing it must be recovered. Allow a different variable e.g., $z$. May use rounded decimals. Accept values for $p$, $q$, $r$ | | **M1** |
| | $32x^2+230x+575=0$ | Correct **equation in $x$**. Three terms in any order on same side. Allow any integer multiple. If just values are given $px^2+qx+r=0$ must be seen in the work | **A1** |
| | | | **(5)** |
| | | | **(Total 8 marks)** |