9. The rectangular hyperbola H has equation $xy = c^2$ where c is a positive constant.

The point $P\left(cp, \frac{c}{p}\right)$, where $p \neq 0$ and $p \neq \pm 1$, lies on H.

(a) Use calculus to show that the normal to H at P has equation

$$p^3x - py = c(p^4 - 1)$$

(4)

Given that

- the point $Q\left(cq, \frac{c}{q}\right)$, where $q \neq 0$ and $p \neq q$, also lies on H
- the normal to H at P meets H again at the point Q

(b) show that

$$p^3q = -1$$

(3)

Given also that the normal to H at Q meets H again at the point R,

(c) show that the gradient of PR is

$$-\frac{1}{p^k}$$

where k is a constant to be determined.

(5)

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Question 9 continued

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(Total for Question 9 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

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