| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| **5** | $3z+\mathrm{i}w=-1+\mathrm{i} \quad kz-w=-9\mathrm{i}$ | | |
| **(a)** | $w=kz+9\mathrm{i} \Rightarrow 3z+\mathrm{i}(kz+9\mathrm{i})=-1+\mathrm{i}$ ; **or**<br>$kz-w=-9\mathrm{i} \Rightarrow \mathrm{i}kz-\mathrm{i}w=9 \Rightarrow \mathrm{i}kz-(-1+\mathrm{i}-3z)=9$<br>$\Rightarrow z(3+k\mathrm{i})=8+\mathrm{i} \Rightarrow z=\dfrac{8+\mathrm{i}}{3+k\mathrm{i}}$ | Uses appropriate algebra to eliminate $w$ and reaches<br>$z=\dfrac{a+ib}{c+\mathrm{i}d}=\mathrm{f}(k)$ | **M1** |
| | $z=\dfrac{8+\mathrm{i}}{3+k\mathrm{i}}\times\dfrac{3-k\mathrm{i}}{3-k\mathrm{i}}$ | Appropriate multiplier seen. Condone "$\times(3-k\mathrm{i})$" if subsequently both numerator and denominator are multiplied. | **dM1** |
| | $z=\dfrac{24+k+(3-8k)\mathrm{i}}{9+k^2}$ * | Fully correct proof. Condone invisible brackets if recovered. | **A1*** |
| **Alt**<br>Sim equations | $z=a+b\mathrm{i}, w=c+d\mathrm{i}$<br>Substitute into given equations:<br>$3(a+b\mathrm{i})+\mathrm{i}(c+d\mathrm{i})=-1+\mathrm{i}$<br>$k(a+b\mathrm{i})-(c+d\mathrm{i})=-9\mathrm{i}$<br>Equate real and imaginary parts, leading to:<br>$3a-d=-1,\ 3b+c=1$<br>$ka-c=0,\quad kb-d=-9$<br>Eliminate $c$ and $d$ leading to:<br>$a=\dfrac{24+k}{9+k^2}$ and $b=\dfrac{3-8k}{9+k^2}$<br>So $z=\dfrac{24+k+(3-8k)\mathrm{i}}{9+k^2}$ * | M1: Obtains four simultaneous equations in the real and imaginary coefficients of $z$ and $w$<br>dM1: Eliminates real and imaginary coefficients of $w$<br>A1*: Fully correct proof. | |
| | | | **(3)** |
| **(b)** | $k=2 \Rightarrow z=\dfrac{26-13\mathrm{i}}{13}=2-\mathrm{i}$<br>$\Rightarrow w=\ldots$ | Substitutes $k=2$ to find $z$ and then substitutes to find $w$. If they substitute $k=2$ into simultaneous equations, they must reach $w=\ldots$ | **M1** |
| | $w=2(2-\mathrm{i})+9\mathrm{i}=4+7\mathrm{i}$ * | Correct proof with sufficient working | **A1*** |
| | | | **(2)** |
| **(c)(i)** | $\dfrac{\lvert w\rvert}{\lvert z\rvert}=\dfrac{\sqrt{4^2+7^2}}{\sqrt{2^2+1^2}}\left\{=\dfrac{\sqrt{65}}{\sqrt{5}}\right\}=\sqrt{13}$ ; **or**<br>$\dfrac{\lvert w\rvert}{\lvert z\rvert}=\left\lvert\dfrac{w}{z}\right\rvert=\left\lvert\dfrac{4+7\mathrm{i}}{2-\mathrm{i}}\right\rvert=\left\lvert\dfrac{1}{5}+\dfrac{18}{5}\mathrm{i}\right\rvert=\sqrt{\left(\dfrac{1}{5}\right)^2+\left(\dfrac{18}{5}\right)^2}=\sqrt{13}$ | M1: Correct use of modulus formula. Award when a correct numerical expression is seen for either modulus.<br>Note: may see $\sqrt{zz^*}$ or $\sqrt{ww^*}$<br>A1: $\sqrt{13}$ only | **M1**<br>**A1** |
| **(c)(ii)** | $\arg(z+w)=\arg(6+6\mathrm{i})$<br>$=\arctan(1)=\dfrac{\pi}{4}$ | $\dfrac{\pi}{4}$ or $45\{^\circ\}$<br>Isw if $\dfrac{\pi}{4}$ is written as a decimal (rounded or truncated to 0.78 or 0.79 to 2 d.p.), but not if a correct answer is followed by e.g. $-\dfrac{\pi}{4}, \dfrac{3\pi}{4}, -45^\circ, 135^\circ$ | **B1** |
| | | | **(3)** |
| | | | **Total 8** |