Question Number	Scheme	Notes	Marks
6(a)	$n=1 \Rightarrow \text{LHS} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^1 = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ $\text{RHS} = \begin{pmatrix} 2(1)+1 & -2(1) \\ 2(1) & 1-2(1) \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$	Shows that LHS and RHS $= \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ or **A** for $n = 1$ At minimum, condone only substitution into RHS, reaching $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ and statement "true" o.e. **Note:** it is possible to use $n = 0$ in the basis case. In this case they would prove LHS=RHS=**I** **Note:** the conclusion for the basis case may be seen as a part of the final conclusion of proof.	**B1**
	Assume true for $n = k$, i.e., $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^k = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}$		
	$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^k \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$ or $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}$ Forms appropriate product of matrices for $\mathbf{A}^{k+1}$		**M1**
	$= \begin{pmatrix} 6k+3-4k & -4k-2+2k \\ 6k+2-4k & -4k-1+2k \end{pmatrix}$ or $\begin{pmatrix} 6k+3-4k & -6k-2+4k \\ 4k+2-2k & -4k-1+2k \end{pmatrix} \Rightarrow \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$ **d**M1: Correct method for multiplication. Look for 3 correct elements unsimplified. A1: Correct simplified matrix with k terms collected. This must be seen, not implied, before being written in terms of $k+1$.		**dM1** **A1**
	$= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$ True for $n = 1$, true for $n = k + 1$ when assumed true for $n = k$, hence true for $n \in \mathbb{N}^+$	Correct working leading to correct matrix in terms of $k+1$ and suitable conclusion. The conclusion must have 3 parts: 1) "True for $n = 1$", but this need not be repeated if seen earlier. 2) True for $n = k \Rightarrow$ true for $n = k+1$, but if "Assume true for $n = k$" is seen earlier, then this need not be repeated, i.e. can conclude with "Hence true for $n = k+1$" o.e. 3) True for $n \in \mathbb{N}^+$, but accept any of these: true for $n \in \mathbb{N}$; true for all n; true for all integers; true for all numbers; true for $n \in \mathbb{R}$; true for all; always true. A1 can follow B0. A0 if work is in terms of n throughout.	**A1**
	Note: A meet in the middle approach is possible. For example, reaching $\begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$ for dM1 A1 and expanding $\begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$ to reach the same.		
			(5)

(b)	$\mathbf{A}^5 = \begin{pmatrix} 11 & -10 \\ 10 & -9 \end{pmatrix}$	Correct $\mathbf{A}^5$, but also give B1 if their calculation of $\mathbf{A}^5$ is unsimplified and/or embedded in their calculation of **C**.	**B1**
	$\mathbf{C} = \mathbf{A}^5 - \mathbf{AB} + 7\mathbf{I} = \begin{pmatrix} 11 & -10 \\ 10 & -9 \end{pmatrix} - \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} p & -2 \\ -4 & 2 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$ $= \begin{pmatrix} 11 & -10 \\ 10 & -9 \end{pmatrix} - \begin{pmatrix} 3p+8 & -10 \\ 2p+4 & -6 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} = \ldots$ A full attempt at **C** with their $\mathbf{A}^5$ including correct method of multiplication for **AB** (with 3 correct elements unsimplified). Must reach a single matrix **C** in terms of p.		**M1**
	$\begin{pmatrix} 10-3p & 0 \\ 6-2p & 4 \end{pmatrix}$	Correct matrix	**A1**
			(3)
(c)	$\left\{ p = 3 \Rightarrow \mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \right\}$ Stretch of scale factor 4 parallel to the y–axis o.e.	M1: If a matrix **C** is written down, it must be of the form $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & q \end{pmatrix}, \lvert q \rvert \neq 1$. States "stretch". Condone "enlarge", but not "multiply". Must be a single transformation. M0 if alternatives also given. A1ft: Fully correct description. Requires "scale factor 4", "4 times" o.e., not e.g. "by 4 units". Accept "y-direction" or "vertically". FT on their $\lvert q \rvert \neq 1$	**M1** **A1ft**
			(2)
			Total 10