6.

$$\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$$

(a) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\mathbf{A}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$$

(5)

$$\mathbf{B} = \begin{pmatrix} p & -2 \\ -4 & 2 \end{pmatrix}$$

where p is a constant.

Given that

$$\mathbf{C} = \mathbf{A}^5 - \mathbf{AB} + 7\mathbf{I}$$

where $\mathbf{I}$ is the 2×2 identity matrix,

(b) use the result in part (a) to show that

$$\mathbf{C} = \begin{pmatrix} 10-3p & 0 \\ 6-2p & q \end{pmatrix}$$

where q is a constant to be determined.

(3)

(c) Describe fully the single transformation represented by $\mathbf{C}$ when $p = 3$

(2)

(Total for Question 6 is 10 marks)