7. The quadratic equation $2x^2 - 3x + 5 = 0$ has roots α and β.

You must answer parts (a) and (b) without solving the quadratic equation.

(a) Show that

$$(\alpha + 1)^3 (\beta + 1)^3 = 125$$

(3)

(b) Determine a quadratic equation with integer coefficients which has roots

$$\frac{1}{(\alpha + 1)^3} \quad \text{and} \quad \frac{1}{(\beta + 1)^3}$$

(5)

(Total for Question 7 is 8 marks)

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8. (a) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2 (n + 1)^2$$

(5)

(b) Hence determine the value of n for which

$$\sum_{r=2n}^{4n-1} r^3 = 1363 \left(\sum_{r=1}^{n} 1 \right)^2$$

(4)

(Total for Question 8 is 9 marks)