Question Number	Scheme	Notes	Marks
8(a)	$n=1 \Rightarrow \text{LHS} = \sum_{r=1}^{1} r^3 = 1^3 = 1$ $\text{RHS} = \frac{1}{4}\times 1^2 \times (1+1)^2 = \frac{1}{4}\times 1\times 2^2 = 1$	Shows that LHS and RHS =1 for n =1. Must be some substitution into RHS. Gives conclusion "true", "shown", "proved", "✓" o.e. **Note:** the conclusion for the basis case may be seen as a part of the final conclusion of proof.	**B1**
	Assume true for $n=k$, i.e., $\sum_{r=1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$		
	$\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$	Uses $\sum_{r=1}^{k+1} r^3 = \left(\sum_{r=1}^{k} r^3\right) + (k+1)$th term	**M1**
	$= (k+1)^2\left(\frac{1}{4}k^2 + k + 1\right) = \frac{1}{4}(k+1)^2(k^2+4k+4)$	Reaches $\frac{1}{4}(k+1)^2$ (quadratic in k) Must reach this intermediate step; it cannot be implied by $\frac{1}{4}(k+1)^2(k+2)^2$	**dM1**
	$= \frac{1}{4}(k+1)^2(k+2)^2$ or $\frac{1}{4}(k+1)^2((k+1)+1)^2$	Either expression	**A1**
	True for n = 1, true for $n = k+1$ when assumed true for $n = k$, hence true for $n \in \mathbb{N}^+$	Fully correct proof with correct conclusion. The conclusion must have 3 parts: 1) "True for $n=1$", but this need not be repeated if seen earlier. 2) True for $n=k \Rightarrow$ true for $n=k+1$, but if "Assume true for $n=k$" is seen earlier, then this need not be repeated, i.e. can conclude with "Hence true for $n=k+1$" o.e. 3) True for $n \in \mathbb{N}^+$, but accept any of these: true for $n \in \mathbb{N}$; true for all n; true for all integers; true for all numbers; true for $n \in \mathbb{R}$; true for all; always true. A1 can follow B0. A0 if work is in terms of n throughout.	**A1**
	Note: A meet in the middle approach is possible. For example, after reaching $\sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$ for M1, expanding brackets to give $\frac{1}{4}k^4 + \frac{3}{2}k^3 + \frac{13}{4}k^2 + 3k + 1$. They must then expand the target expression $\frac{1}{4}(k+1)^2(k+2)^2$ to a quartic (dM1) to reach the same expression (A1), followed by conclusion as above (A1).		
			(5)

(b)	$\sum_{r=2n}^{4n} r^3 = \frac{1}{4}(4n)^2(4n-1)^2 - \frac{1}{4}(2n)^2(2n-1)^2$	Attempts $f(4n-1) - f(2n-1)$	**M1**
	$\sum_{r=1}^{n} 1 = n$	Uses $\sum_{r=1}^{n} 1 = n$	**B1**
	$4n^2(16n^2-8n+1) - n^2(4n^2-4n+1) = 1363n^2$ $\Rightarrow 60n^2 - 28n - 1360 = 0$ $\Rightarrow 15n^2 - 7n - 340 = 0$ $\Rightarrow (15n+68)(n-5) = 0 \Rightarrow n = 5$	M1: Reaches 3TQ in n that arises from at least one application of the sum of cubes formula, and solves (usual rules). Alternatively, solves a quartic to reach a value for n. In both cases, must reach a positive value of n. A1: $n = 5$ only	**M1** **A1**
			(4)
			Total 9